Quiz 6 Solutions

Tuesday, July 07, 2009
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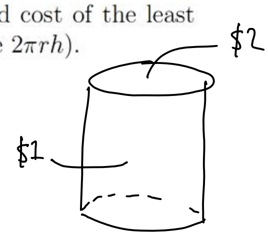
**Math 175** **Quiz 6** **Name:**

Due Monday at the beginning of class. Do your own work. You may refer to your notes or book, but you MUST work independently. Give mathematical justification for your answers.

1. Your company needs to design cylindrical metal containers with a volumne of 16 cubic feet. The top and bottom will be made of a sturdy material that costs \$2 per square foot, while the material for the sides costs \$1 per square foot. Find the radius, height, and cost of the least expensive container. (Area of a disk is $\cancel{2\pi r}$ $\pi r^2$ while the area of the side will the $2\pi rh$).




Cost = \$1 * area of side + \$2 * area of ends

$= 1 * 2\pi rh + 2 * 2 * \pi r^2$

$= 2\pi rh + 4\pi r^2$

$V = \pi r^2 h = 16$

$\Rightarrow h = 16/\pi r^2$

$$C(r) = 2\pi r \cdot \frac{16}{\pi r^2} + 4\pi r^2, \quad r > 0$$

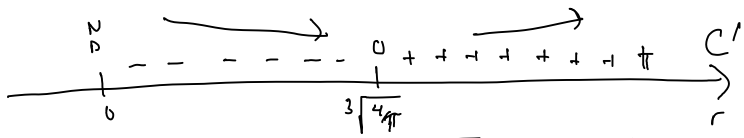
$$= \frac{32}{r} + 4\pi r^2$$

$$C'(r) = \frac{-32}{r^2} + 8\pi r = \frac{8\pi r^3 - 32}{r^2}$$

undefined if $r = 0$

$= 0$ if $r^3 = \frac{32}{8\pi} = \frac{4}{\pi} \Rightarrow r = \sqrt[3]{\frac{4}{\pi}}$

$r \approx 1.0839$ ft



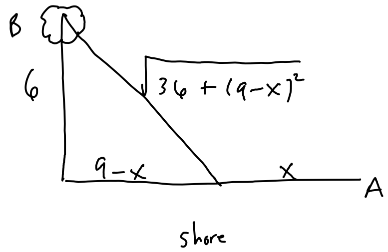

cost has a global min. at $r = \sqrt[3]{4/\pi} \approx 1.0839$ ft

$h = \frac{16}{\pi\left(\sqrt[3]{4/\pi}\right)^2} \approx 4.3354$ ft

(can check that $\pi r^2 h \approx 16$ !!)

& the minimum cost is $C(r) = C\left(\sqrt[3]{4/\pi}\right) = 44.29$ dollars

2. A company wishes to run a utility cable from point $A$ on the shore (see figure) to an installation at point $B$ on the island. The island is 6 miles from shore. It costs \$400 per mile to run the cable on land and \$500 per mile underwater. Assume the cable starts at $A$ and runs along the shoreline, then angles and runs underwater to the island. Find the point at which the line should begin to angle to yield the minimum cost.




$$\text{Cost}(x) = 500\sqrt{36+(9-x)^2} + 400x, \quad x \in [0,9]$$

$$C'(x) = 400 + 500 \cdot \cancel{\tfrac{1}{2}}\left(36+(9-x)^2\right)^{-1/2} \cancel{2}(9-x)'(-1)$$

$$= 400 + \frac{500(x-9)}{\sqrt{36+(9-x)^2}} = \frac{400\sqrt{36+(9-x)^2} + 500(x-9)}{\sqrt{36+(9-x)^2}} = 0$$

$$\Rightarrow \quad 400\sqrt{36+(9-x)^2} + 500(x-9) = 0$$

$$\cancel{400}^{4}\sqrt{36+(9-x)^2} = \cancel{500}^{5}(9-x)$$

$$16\left(36+(9-x)^2\right) = 25(9-x)^2$$

$$16(36+81-18x+x^2) = 25(81-18x+x^2)$$

$$16(117-18x+x^2) = 25(81-18x+x^2)$$

$$0 = 153 - 162x + 9x^2$$

$$0 = 17 - 18x + x^2$$

$$0 = (x-17)(x-1)$$

$$x=1,\ x=17 \quad \text{critical values}$$

on $[0,9]$ possible locations for absolute min. are

| $x$ | $C(x) = 500\sqrt{36+(9-x)^2} + 400x$ |
|---|---|
| 0 | \$5408.30 |
| 1 | \$5400.00 ← absolute min cost. |
| 9 | \$6600 |

, build 1 mile along shore & underwater from there to island.