Quiz 5 Solutions

Math 175 **Quiz 5** **Name:** Solutions

1. Consider the following function $g(x) = \dfrac{3-2x}{x^2}$

(a) Analyze g (intercepts, domain, asymptotes, etc.)

domain: $x \neq 0$

y-int: none

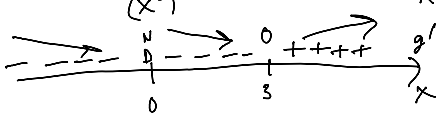
x-int: $g(x)=0 \Rightarrow 3-2x=0 \Rightarrow x=1.5$

VA: $x=0$, HA: $\lim_{x\to\infty} \dfrac{3-2x}{x^2} = \lim_{x\to\infty} \dfrac{(3-2x)\,{}^1/_{x^2}}{x^2/x^2} = \lim_{x\to\infty} \dfrac{3/x^2 - 2/x}{1} = \dfrac{0-0}{1} = 0$

HA: $y=0$

(b) Analyze g' (increasing, decreasing, local extrema)

$$g'(x) = \frac{x^2(-2)-(3-2x)(2x)}{(x^2)^2} = \frac{-2x^2-6x+4x^2}{x^4} = \frac{2x^2-6x}{x^4} = \frac{2x(x-3)}{x^4} = \frac{2(x-3)}{x^3}$$

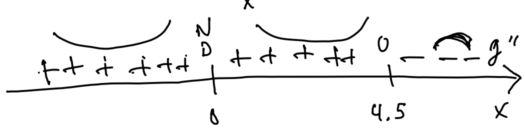


g has a local min at $x=3$

$y = g(3) = \dfrac{3-2(3)}{3^2} = \dfrac{-3}{3^2} = -\dfrac{1}{3}$

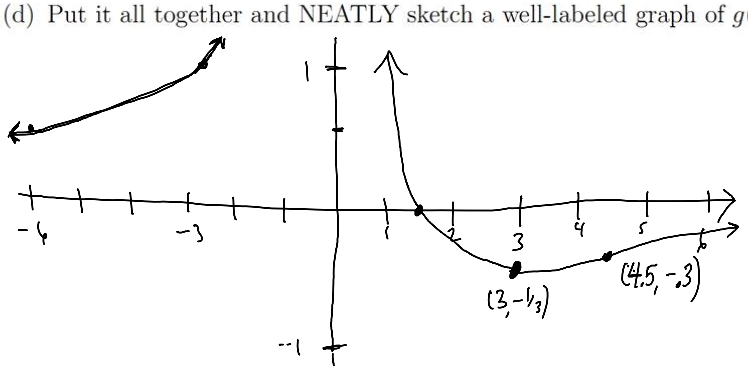
(c) Analyze g'' (concavity, inflection points)

$$g''(x) = \frac{x^3(2) - 2(x-3)3x^2}{x^6} = \frac{2x^3-6x^3+18x^2}{x^6} = \frac{18x^2-4x^3}{x^6} = \frac{2\cancel{x^2}(9-2x)}{\cancel{x^6}\,x^4}$$



inflection point at $x=4.5$, $y = g(4.5) = \dfrac{3-2(4.5)}{(4.5)^2} \approx -.30$

(d) Put it all together and NEATLY sketch a well-labeled graph of $g(x)$



$g(-3) = \dfrac{3-2(-3)}{(-3)^2} = \dfrac{9}{9} = 1$

$g(-6) = \dfrac{3-2(-6)}{(-6)^2} = \dfrac{15}{36} \approx .42$

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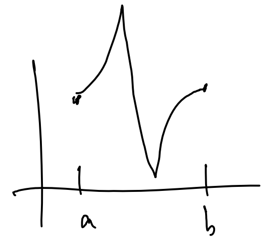
2. Find the absolute extrema of $f(x) = x^4 - 8x^2 + 16$ on the interval $[-1, 3]$.

$f'(x) = 4x^3 - 16x = 4x(x^2-4) = 4x(x-2)(x+2) \rightarrow$ critical values $x = 0, -2, 2$

Only C.V.'s 0 & 2 are in the interval



Check for absolute extrema at CVs & endpoints

x	$y = f(x) = x^4 - 8x^2 + 16$
-1	$(-1)^4 - 8(-1)^2 + 16 = 1 - 8 + 16 = 9$
0	16
2	$16 - 32 + 16 = 0$ ← abs. min. of 0 at $x = 2$
3	$81 - 72 + 16 = 25$ ← abs. max of 25 at $x = 3$

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