Quiz 3 Solutions

Math 175 **Quiz 4** **Name:** Solutions

1. Use the definition of the derivative to find $f'(x)$ if $f(x) = 4 + \frac{4}{x}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\left(4+\frac{4}{x+h}\right) - \left(4+\frac{4}{x}\right)}{h} = \lim_{h\to 0} \left(\frac{4}{x+h} - \frac{4}{x}\right)\cdot\frac{1}{h}$$

$$= \lim_{h\to 0} \left(\frac{4}{x+h}\cdot\frac{x}{x} - \frac{4}{x}\cdot\frac{x+h}{x+h}\right)\cdot\frac{1}{h} = \lim_{h\to 0} \left(\frac{4x}{(x+h)x} - \frac{4x+4h}{(x+h)x}\right)\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-4\cancel{h}}{x(x+h)}\cdot\frac{1}{\cancel{h}} = -\frac{4}{x^2}$$

2

2. Verify that you did the previous problem correctly by using the differentiation shortcuts.

$$f(x) = 4 + 4x^{-1} \Rightarrow f'(x) = 0 + 4(-1)x^{-2} = -\frac{4}{x^2}$$ ✓

1

3. $f(x) = x^2 + x$ $\qquad f'(x) = 2x+1$

(a) Find the slope of the tangent line at $(1, f(1))$.

$$m = f'(1) = 2(1) + 1 = 3$$

1

(b) Find the equation of the tangent line at $(1, f(1))$.

$$x_0 = 1, \quad y_0 = f(x_0) = f(1) = 2$$

$$y - y_0 = m(x - x_0)$$

$$y - 2 = 3(x-1)$$

$$y = 3x - 3 + 2$$

$$y = 3x - 1$$

1

4. If $h(t) = \frac{-t^2}{2t+1}$, find and simplify $h'(t)$.

$$h'(t) = \frac{(2t+1)(-2t) - (-t^2)(2)}{(2t+1)^2} = \frac{-4t^2-2t+2t^2}{(2t+1)^2} = \frac{-2t^2-2t}{(2t+1)^2}$$

$$= \frac{-2t(t+1)}{(2t+1)^2}$$

2

5. The total sales of a company (in millions of dollars) t months from now are given by $S(t) = 0.015t^4 + 0.5t^3 + 3.4t^3 + 10t - 3$.

(a) Find $S(4)$ and explain what it means.

$S(4) = 290.44$ million dollars

→ Total sales 4 months from now will be 290.44 million dollars

1

(b) Find $S'(4)$ and explain what it means.

$S'(t) = .06t^3 + 1.5t^2 + 10.2t^2$

$S'(4) = 201.04$ million dollars/month

2

Total sales 4 months from now will be increasing at 201.04 million dollars per month.