Math 175 **Quiz 3** **Name:**

1. Let $f(x) = \begin{cases} \dfrac{x^2-9}{x+3} , & \text{if } x < 0 \\ , & \\ \dfrac{x^2-9}{x-3} , & \text{if } x > 0 \end{cases}$

Find each indicated limit or explain why it does not exist:

(a) $\lim_{x\to -3} f(x) = \lim_{x\to -3} \frac{x^2-9}{x+3} = \lim_{x\to -3} \frac{(x-3)\cancel{(x+3)}}{\cancel{x+3}} = -3-3 = -6$

$\left(\text{"}\frac{0}{0}\text{"}\right)$

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(b) $\lim_{x\to 0} f(x)$ function changes rules at $x=0$ so

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{x^2-9}{x+3} = \frac{-9}{3} = -3$

& $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x^2-9}{x-3} = \frac{-9}{-3} = +3$

since the left & right limits are different $\lim_{x\to 0} f(x)$ does not exist

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2. For the function $f(x) = 1+\sqrt{x}$ compute the following limit: $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$

$$= \lim_{h\to 0} \frac{\cancel{1}+\sqrt{2+h} - (\cancel{1}+\sqrt{2})}{h} = \lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h} \cdot \frac{\sqrt{2+h}+\sqrt{2}}{\sqrt{2+h}+\sqrt{2}} = \lim_{h\to 0} \frac{\overset{1}{(\cancel{2}+\cancel{h})-\cancel{2}}}{\cancel{h}(\sqrt{2+h}+\sqrt{2})}$$

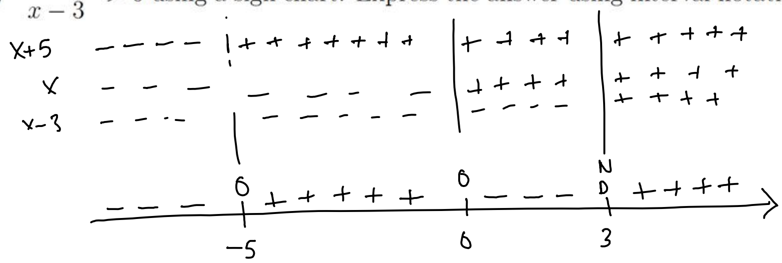
$$= \lim_{h\to 0} \frac{1}{\sqrt{2+h}+\sqrt{2}} = \frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

limit symbol no longer necessary!

2

3. Solve the inequality $\dfrac{x^2+5x}{x-3} > 0$ using a sign chart. Express the answer using interval notation.

$$\frac{(x+5)x}{x-3} > 0$$



$x \in (-5,0) \cup (3,\infty)$

2

4. Let $f(x) = \begin{cases} \dfrac{x-2}{x^2-4} & , \quad \text{if } x \neq 2 \\ & , \\ 1 & , \quad \text{if } x = 2 \end{cases}$

Is f continuous at $x = 2$? Carefully explain why or why not using the definition of continuity.

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{x-2}{x^2-4} = \lim_{x\to 2} \frac{\cancel{x-2}}{\cancel{(x-2)}(x+2)} = \frac{1}{2+2} = \frac{1}{4}$$

but $f(2) = 1$ so $\lim_{x\to 2} f(x) \neq f(2)$

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Therefore $f(x)$ is not continuous at $x=2$!