

5. A diver jumps off a diving board into a swimming pool. Her height f (in feet) above the water t seconds after she jumps is given by $h(t) = -16t^2 + 20t + 6$.

(a) Find her average rate of change (velocity) from $t = 0.5$ seconds to $t = 1$ second.

(b) Find her instantaneous rate of change when $t = 1$.

(c) When does she hit the water?

(d) How fast is she moving when she hits the water?

6. Evaluate the following limits algebraically or show that they do not exist:

(a) $\lim_{x \rightarrow -4} \frac{-x}{(x-4)^2}$

(b) $\lim_{x \rightarrow 0} \frac{x}{|x|}$

(c) $\lim_{x \rightarrow 3^-} \frac{2x+5}{x^2-9}$

(d) $\lim_{x \rightarrow 3} \frac{x-3}{x^2-x-6}$

7. Consider the function

$$f(x) = \begin{cases} 3x - 2 & \text{for } x < 2 \\ c & \text{for } x = 2 \\ 8 - 2x & \text{for } x > 2 \end{cases}$$

Can you find a value of c that makes $f(x)$ a continuous function on all real numbers? Justify your answer either way (using the definition of continuity).

8. Write the equation of the line that is tangent to $f(x) = x^2 - 3$ at the point at which $x = 1$.

9. Find all values of x for which the graph of $p(x) = \frac{x^2}{(5x - 1)^4}$ has a horizontal tangent line.

10. The salvage value S (in dollars) of a company airline after t years is estimated to be given by $S(t) = 300,000e^{-0.3t}$. What is the rate of depreciation (in dollars per year) after 5 years?

11. Find the derivative of the given function. Simplify before differentiating whenever possible, but DO NOT simplify the derivative.

(a) $r(x) = -32x^2 + 400x + 500$

(b) $q(x) = \sqrt[3]{x}(3x^4 + 2x^2)$

(c) $g(x) = \ln[(x^2 + 5x)^6]$

(d) $f(x) = e^{\sqrt{x^3+5x}}$

12. Sketch a function $f(x)$ for which all of the following are true:

- $f(1) = -1, f(-1) = 2$
- $\lim_{x \rightarrow 1^-} f(x) = 1$
- f is continuous at $x = -1$
- f is NOT differentiable at $x = -1$
- $f'(x) = 2$ for $x > 1$

