6.4

Tuesday, July 14, 2009
7:41 AM

Pg 393/49 $\int x(x^3+2)^2\,dx = \int x\left(x^6+4x^3+4\right)dx = \int\left(x^7+4x^4+4x\right)dx$

$= \frac{1}{8}x^8 + \frac{4}{5}x^5 + 2x^2 + C$

~~$u = x^3+2$~~
~~$du = 3x^2\,dx$~~
~~NOT USEFUL~~

Pg 393/57 $\int \frac{1}{x^2} e^{-1/x}\,dx = \int e^u\,du = e^u + C = e^{-1/x} + C$

$u = \frac{-1}{x} = -x^{-1}$

$du = x^{-2}\,dx = \frac{1}{x^2}\,dx$

Type I

$u = e^{-1/x}$

$du = e^{-1/x} \cdot \frac{1}{x^2}\,dx$

$\int \underbrace{\frac{1}{x^2}\underbrace{e^{-1/x}}_{u}\,dx}_{du} = \int du = \int 1\,du = u + C$

$= e^{-1/x} + C$

---

(65) $\int e^{au}\,du = \int e^v \cdot \frac{1}{a}\,dv = \frac{1}{a}\int e^v\,dv = \frac{1}{a}e^v + C$

$= \frac{1}{a}e^{au} + C$

$v = au$

$dv = a\,du$

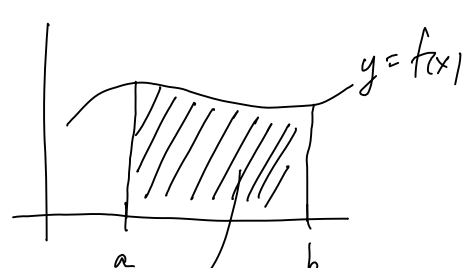
$\frac{1}{a}\,dv = du$

---

§6.4 Area & Definite Integrals (Consumer/Producer surplus, useful life)

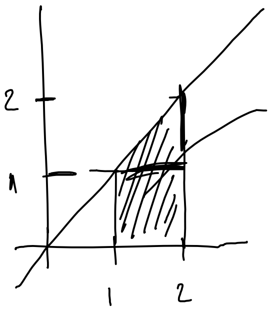
Notation $\int_a^b f(x)\,dx$ } definite integral

↑ limits of integration (a and b)



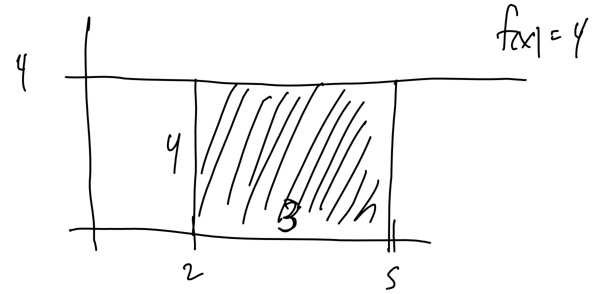
represents the shaded area

$f(x) = x$



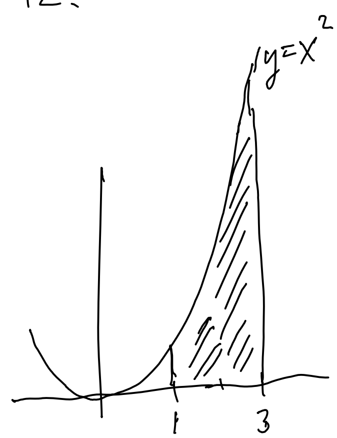
Area $= \int_1^2 f(x)\,dx = \int_1^2 x\,dx = \frac{3}{2}$

$\int_2^5 4\,dx = 12.$



$\int_1^3 x^2\,dx$

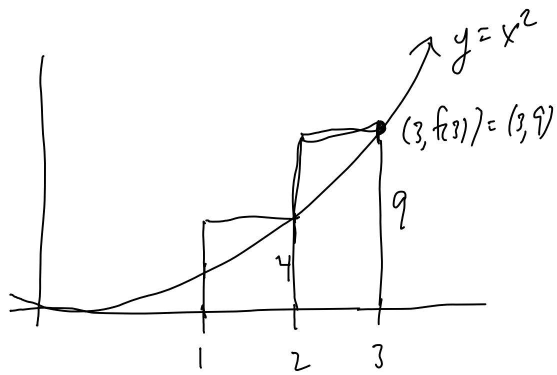
$= ?$



<u>IDEA</u> approx. w/ rectangles.

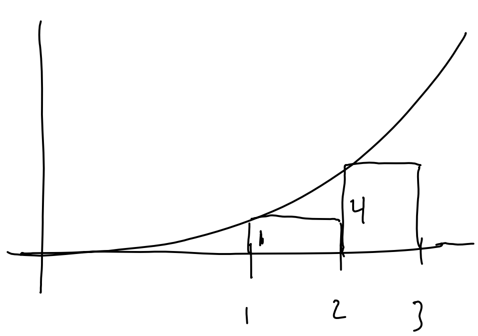
(1)

too big.



Right endpoint $A_R = wh + wh$

$= 1 \cdot 4 + 1 \cdot 9 = 13.$



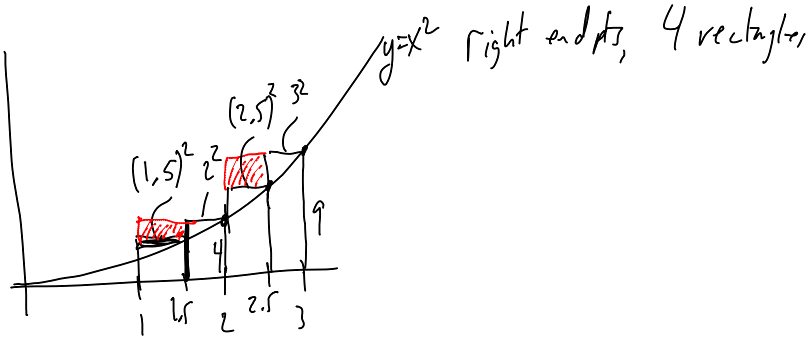
Left end pt (Too small)

$A_L = 1 \cdot 1 + 1 \cdot 4$

$= 5$

a better guess would be $\frac{A_R + A_L}{2} = \frac{13+5}{2} = 9$

use more rectangles for better approximations



$A_R = .5(1.5)^2 + .5(2)^2 + .5(2.5)^2 + .5(3)^2$

$= 10.75$

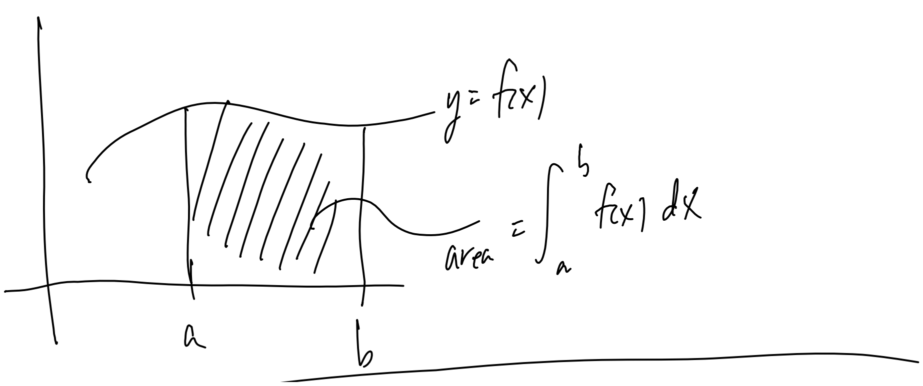
w/ 4

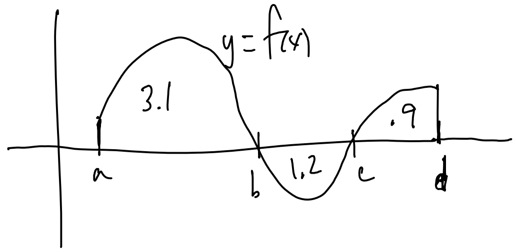
$A_L = 6.75$

$$6.75 \le \int_1^3 x^2\,dx \le 10.75$$

The process continuous adding rectangles until

$$\int_a^b f(x)\,dx = \lim_{\#\text{ of rectangles} \to \infty} A_R = \lim_{\#\text{ of rect.} \to \infty} A_L$$





$$\int_a^d f(x)\,dx = \text{net area}$$

$= \overline{\text{area above x-axis}}$

area below x-axis

$= 3.1 - 1.2 + .9$

$= 2.8$

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

$$= 3.1 - 1.2 = 1.9$$

Rule → $$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx = -3.1$$

Know $\int_1^4 x\,dx = 7.5$, $\int_1^4 x^2\,dx = 21$, $\int_4^5 x^2\,dx = \frac{61}{3}$

30. $$\int_1^4 3x^2\,dx = 3\int_1^4 x^2\,dx = 3(21) = 63$$

32. $$\int_1^4 (7x - 2x^2)\,dx = 7\int_1^4 x\,dx - 2\int_1^4 x^2\,dx$$

$$= 7(7.5) - 2(21) = 10.5$$

36. $$\int_1^5 -4x^2\,dx = -4\int_1^5 x^2\,dx = -4\left[\int_1^4 x^2\,dx + \int_4^5 x^2\,dx\right]$$

$$= -4\left[21 + 61/3\right] = -4\left[124/3\right] = \frac{-496}{3}$$

38. $\int_5^5 (10-7x+x^2)\,dx = 0$

40. $-\int_1^4 x(1-x)\,dx = -\int_1^4 (x-x^2)\,dx = -\left(\int_1^4 x\,dx - \int_1^4 x^2\,dx\right)$

$= \int_4^1 x(1-x)\,dx$

$= -(7.5 - 21 =) + 13.5$

$V'(t) = \frac{15}{t}$, $V(1) = 15$, what is $V(4) =$

$\hookrightarrow V(t) = 15\ln|t| + C$

$V(1) = 15\ln|1| + C = 15$

$15(0) + C = 15$

$C = 15$

$V(t) = 15\ln|t| + 15$

$\therefore \quad V(4) = 15\ln|4| + 15$