6.1-6.2

Monday, July 13, 2009
7:18 AM

§ 6.1 Antiderivatives

$f(x) \longrightarrow F(x)$ where $F(x)$ is the general anti D. of f.

meaning $F'(x) = f(x)$

$$\int f(x)\,dx = F(x) + C$$

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + C}$$

ex.

$$\int \left(5x^3 - \sqrt{x} + \frac{3}{x^2}\right)dx = \int \left(5x^3 - x^{1/2} + 3x^{-2}\right)dx$$

$$= 5\frac{x^4}{4} - \frac{x^{3/2}}{3/2} + \frac{3x^{-1}}{-1} + C$$

$$= \frac{5}{4}x^4 - \frac{2}{3}x^{3/2} - \frac{3}{x} + C$$

$$\boxed{\int \frac{1}{x}\,dx = \ln|x| + C} \qquad \boxed{\int e^x\,dx = e^x + C}$$

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad n \neq -1}$$

6.1/#62

$$\int \frac{(1+z^2)^2}{z}\,dz = \int \frac{1+2z^2+z^4}{z}\,dz = \int \frac{1}{z} + 2z + z^3\,dz$$

$$= \ln|z| + \cancel{2}\frac{z^2}{\cancel{2}} + \frac{z^4}{4} + C$$

#66 $\underbrace{\frac{dy}{dx} = 5 - 4x}_{\text{given } y'}$, $\underbrace{y(0) = 20}$

we want find $y(x)$ so that $y(0) = 20$

$$y(x) = \int y'(x)\,dx = \int (5-4x)\,dx = 5x - \frac{4x^2}{2} + C = 5x - 2x^2 + C$$

$$y(x) = 5x - 2x^2 + C$$

but we want $y(0) = 20$

$$y(0) = 5(0) - 2(0)^2 + C = C$$

use $C = 20$

now $y(x) = 5x - 2x^2 + 20$

HW
finish 6.1

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§6.2 Integration by substitution

chain rule reminder:

$$\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)$$

$$\frac{d}{dx}\, out(in(x)) = out'(in(x))\, in'(x)$$

as an antiD. rule:

$$\int f'(g(x))\, g'(x)\,dx = f(g(x)) + C$$

Ex. $$\int 3(x^2+1)^2\, 2x\,dx = (x^2+1)^3 + C$$

u-substitution,

$$\int \overbrace{(x^2+1)^2}^{u^2} \quad \int u^2 \cdot 3\,du = \int 3u^2\,du$$

$$\int \underbrace{6x\,(x+1)\,dx}_{3\,du} = \int$$

$u = x^2+1$

$du = 2x\,dx$

$3\,du = 6x\,dx$

$u = g(x)$

$\underbrace{du = g'(x)\,dx}_{\text{differential}}$

$$\boxed{\int f'(\underbrace{g(x)}_{u})\,\underbrace{g'(x)\,dx}_{du} = \int f'(u)\,du = f(u)+C = f(g(x))+C}$$

$$\frac{\cancel{3}u^3}{\cancel{3}} + C = (x^2+1)^3 + C$$

<u>Ex</u>

$$\int \underbrace{e^{-x^2}}_{e^u}\,\underbrace{x\,dx}_{-\frac{1}{2}du} = \int e^u \cdot\left(\frac{-1}{2}\right)du = -\frac{1}{2}\int e^u\,du$$

$u = -x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$$= \frac{-1}{2}e^u + C$$

$$= \frac{-1}{2}e^{-x^2} + C$$

<u>Type II substitution</u>

u = "inside" function

du = almost exactly right, but needs a small adjustment

(Type I : du = exactly right)

<u>#22</u>

$$\int \frac{x\,dx}{1+x^2} = \int \frac{\frac{1}{2}du}{u} = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|1+x^2| + C$$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

#30 Type III $u =$ inside, $du = \ldots$ } allows algebra that may lead to solution

WARM UP:

$$\int (x+9)\sqrt{x}\,dx = \int (x+9)x^{1/2}\,dx = \int \left(x^{3/2} + 9x^{1/2}\right)dx$$

$$= \frac{2}{5}x^{5/2} + 9\cdot\frac{2}{3}x^{3/2} + C = \frac{2}{5}x^{5/2} + 6x^{3/2} + C$$

$$\int \underbrace{x}_{u+9}\underbrace{\sqrt{x-9}}_{\sqrt{u}}\underbrace{dx}_{du} = \int (u+9)\sqrt{u}\,du = \int (u+9)u^{1/2}\,du$$

$u = x-9 \rightarrow x = u+9$

$du = dx$

$$= \int \left(u^{3/2} + 9u^{1/2}\right)du$$

$$= \frac{2}{5}u^{5/2} + 6u^{3/2} + C$$

$$= \frac{2}{5}(x-9)^{5/2} + 6(x-9)^{3/2} + C$$

(32.) $$\int \frac{x}{\sqrt{x+5}}\,dx = \int \frac{u-5}{\sqrt{u}}\,du = \int \frac{u}{\sqrt{u}} - \frac{5}{\sqrt{u}}\,du$$

Type III (reuse the substitution)

$u = x+5 \rightarrow x = u-5$

$du = dx$

$$= \int \left(u^1u^{-1/2} - 5u^{-1/2}\right)du$$

$$= \int \left(u^{1/2} - 5u^{-1/2}\right)du$$

$$= \frac{u^{3/2}}{3/2} - \frac{5u^{1/2}}{1/2} + C$$

$$= \frac{2}{3}(x+5)^{3/2} - 10(x+5)^{1/2} + C$$

[34] $$\int x(x+6)^8\,dx \longrightarrow = \int (u-6)u^8\,du$$

$u = x+6$ $\quad x = u-6$

$du = 1\,dx$

$$= \int u^9 - 6u^8\,du$$

$$= \frac{u^{10}}{10} - \frac{6u^9}{9} + C$$

$$= \frac{(x+6)^{10}}{10} - \frac{2}{3}(x+6)^9 + C$$

(20) $\int e^{-.01x}\,dx = \int -100 e^{u}\,du$

$u = -.01x$ $\quad = -100e^{u} + C$

$du = -.01\,dx$ $\quad = -100\, e^{-.01x} + C$

$\frac{1}{-.01}\,du = dx$ $\quad = \frac{1}{-.01} e^{-.01x} + C$

$-100\,du = dx$

$\int e^{2x}\,dx = \frac{1}{2}e^{2x} + C$

$\int e^{\alpha x}\,dx = \frac{1}{\alpha}e^{\alpha x} + C$

#70 $R'(x) = 40 - .02x + \frac{200}{x+1}$, $\quad R(0) = 0$

want $R(x)$ & $R(1000)$

$\int \frac{1}{x+1} = \int \frac{1}{u}\,du$
$= \ln|u| + C$
$= \ln|x+1| + C$

$R(x) = \int R'(x)\,dx = \int 40 - .02x + \frac{200}{x+1}\,dx$

$= 40x - \frac{.02x^2}{2} + 200\int \frac{1}{x+1}\,dx$

$u = x+1$
$du = dx$

$R(x) = 40x - .01x^2 + 200\ln|x+1| + C \leftarrow$ what is C?

$R(0) = \cancel{40(0)}^{0} - \cancel{.01(0)^2}^{0} + \cancel{200\ln|0+1|}^{0} + C = 0 \rightarrow C = 0$

$R(x) = 40x - .01x^2 + 200\ln|x+1|$

Now eval. $R(1000)$

HW incl. 6.2