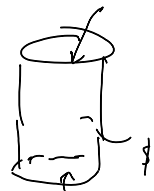
Quiz 6 Solutions + 5.6

Tuesday, July 07, 2009
7:15 AM

Quiz 6 #1



$V = \pi r^2 h = 16 \text{ ft}^3 \to h = \dfrac{16}{\pi r^2}$

C = area of side * \$1 + area of ends * \$2

$= 1 \cdot 2\pi r h + 2\pi r^2 \cdot 2$

$= 2\pi r h + 4\pi r^2$

$= \cancel{2\pi r} \cdot \dfrac{16}{\cancel{\pi r^2}} + 4\pi r^2$

$C(r) = \dfrac{32}{r} + 4\pi r^2, \quad r > 0$

$C'(r) = -\dfrac{32}{r^2} + 8\pi r = \dfrac{-32 + 8\pi r^3}{r^2}$

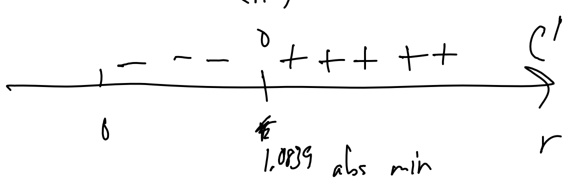
$C'(r)$ undef if $r = 0$

$C'(r) = 0$ if $8\pi r^3 = 32$

$\pi r^3 = 4$

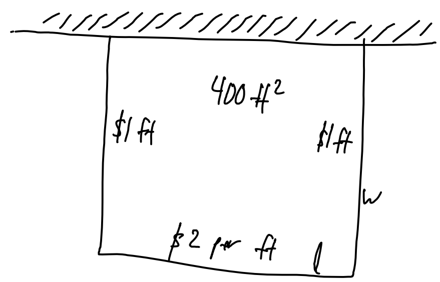
$r^3 = \dfrac{4}{\pi}$

$r = \left(\dfrac{4}{\pi}\right)^{1/3} \approx 1.0839$ ft.



Still to do: find h & C

minimize cost



$A = lw = 400 \to w = \dfrac{400}{l}$

C = length of front * \$2 + length of sides * \$1

$= l \cdot 2 + 2w \cdot 1$

$= 2l + 2 \cdot \dfrac{400}{l}$

$C(l) = 2l + \dfrac{800}{l}, \quad l > 0$

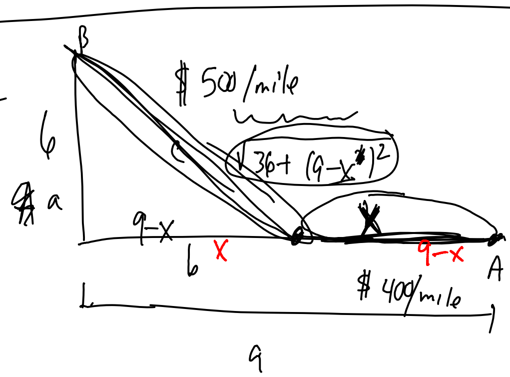
$C'(l) = 2 - \dfrac{800}{l^2}$

$= \dfrac{2l^2 - 800}{l^2} =$

$= 2(l^2 - 400)$

$= \dfrac{2(l-20)(l+20)}{l^2}$

#2



$a^2+b^2=c^2$

$6^2+(9-x)^2=c^2$

$c=\sqrt{36+(9-x)^2}$

$=\sqrt{36+81-18x+x^2}$

$=\sqrt{117-18x+x^2}$

Total Cost = 400 * distance on land + 500 * distance in H_2O

$$C(x) = 400x + 500\sqrt{117-18x+x^2} \qquad x\in[0,9]$$

$$C'(x) = 400 + 500\,\tfrac{1}{2}(117-18x+x^2)^{-1/2}(-18+2x)$$

$$= 400 + \frac{500(2x-18)}{2\sqrt{117-18x+x^2}} = 400+\frac{500(2x-18)}{2\sqrt{36+(9-x)^2}}$$

$$=\frac{400\cdot 2\sqrt{36+(9-x)^2} + 500(2x-18)}{2\sqrt{36+(9-x)^2}} = 0$$

$$8\cancel{00}\sqrt{36+(9-x)^2} + 5\cancel{00}(2x-18)=0$$

$$8\sqrt{36+(9-x)^2} = -5(2x-18)$$

$$64(36+(9-x)^2) = 25(2x-18)^2$$

$$64(36+81-18x+x^2) = 25(4x^2-72x+324)$$

$$2304+5184-1152x+64x^2 = 100x^2-1800x+\cancel{8125}\,8100$$

0 methods $0 = 36x^2-648x+\cancel{637}\,612$

$0 = x^2-18x+17$

Closed Interval: ~~$x = Q7$~~ ...

$0 = x^2 - 18x + 17$

$0 = (x-1)(x-17)$

$x = 1$ or $x = 17$

x	$C(x)$
0	←
1	←
9	←

§3.5/A/pg 355

$e^y = x^2 + y^2$; $(1, 0)$

$(y)^2$

$$\frac{d}{dx} e^y = \frac{d}{dx}(x^2) + \frac{d}{dx} y^2$$

$$e^y y' = 2x + 2(y)^1 y'$$

Solve for y' then sub.

sub. x, y, then solve for y'

$$e^y y' - 2y y' = 2x$$

$$(e^y - 2y) y' = 2x$$

$$y' = \frac{2x}{e^y - 2y}$$

$$y' = \frac{2(1)}{e^0 - 2(0)} = \frac{2}{1-0} = 2$$

$$e^0 y' - 2(0) y' = 2(1)$$

$$y' = 2$$

29. $y^2 - xy - 6 = 0$; $x = 1$ eqn(s) of tangent lines

solve for y(s) (plug in $x=1$)

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3$ OR $y = -2$

TWO pts on the curve w/ $x = 1$

(1, 3) & (1, -2)

2 tangent lines

Now formula for y'

$\frac{d}{dx}$ $2yy' - (xy' + 1y) - 0 = 0$

PR

$2yy' - xy' - y = 0$

$(2y - x)y' = y$

$y' = \frac{y}{2y-x}$

Slope for (1,3)

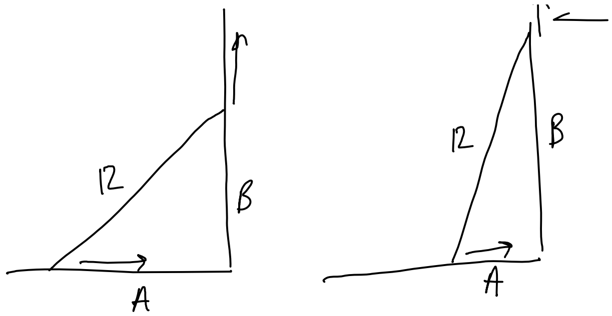
$y' = \frac{3}{2(3)-1} = \frac{3}{5}$

Now build line using

$y - y_0 = m(x - x_0)$

§5.6 Related Rates

A 12 ft ladder is leaning against a wall and the bottom of the ladder is being pushed in at 1 ft/sec. How fast is the top moving up the wall when the bottom is 6 ft from the wall? [see pg 358.



(for STEPS)

$$\frac{dA}{dt} = -1 \ \text{ft/sec}$$

↑ important!

Step 3

want to know $\frac{dB}{dt}$ when

$A = 6$ ft.

(can get B from right triangle!)

Step 4 $\quad A^2 + B^2 = 12^2$

Step 5 $\quad \frac{d}{dt}\left(A^2 + B^2\right) = \frac{d}{dt}\left(12^2\right)$

$$\cancel{2}A\frac{dA}{dt} + \cancel{2}B\frac{dB}{dt} = 0$$

Diff First

Then

Subs.

$$6(-1) + \sqrt{108}\,\frac{dB}{dt} = 0$$

get B

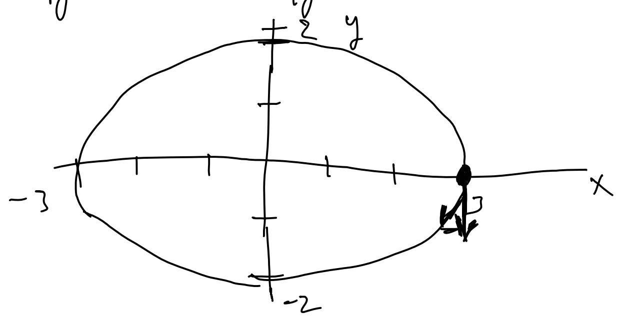
$$A^2 + B^2 = 12^2$$
$$6^2 + B^2 = 12^2$$
$$36 + B^2 = 144$$
$$B^2 = 108$$
$$B = \sqrt{108}$$

$$\frac{dB}{dt} = \frac{+6}{\sqrt{108}} \ \frac{\text{ft}}{\text{sec}}$$

$$\approx .58 \ \text{ft/sec}$$

$\boxed{\S 5.6:\ A, 7, 9, 11, 13, 17, 19, 21, 25, 27}$

#8 pg 361 $4x + 9y = 36$ @ $(3, 0)$ y coordinate decs at 2 units per sec how is x coord chg.



$$\frac{dy}{dt} = -2$$

when $\frac{dx}{dt} = ?$ when $x = 3$, $y = 0$

$$\frac{d}{dt}\left(4x^2 + 9y^2\right) = \frac{d}{dt}(36)$$

$$4 \cdot 2x\frac{dx}{dt} + 9 \cdot 2y\frac{dy}{dt} = 0$$

$$8x\frac{dx}{dt} + 18y\frac{dy}{dt}$$

$$8(3)\frac{dx}{dt} + \cancel{18(0)(2)}^{0} = 0$$

$$\frac{dx}{dt} = 0$$