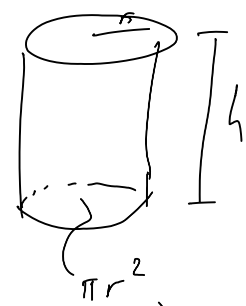
5.3,5.5

Monday, July 06, 2009
7:23 AM

Quiz Prob 1. $V = 16 \text{ ft}^3$

$\pi r^2 h = 16$



$V = \pi r^2 h$

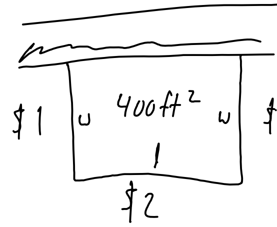
minimize cost

Cost = cost of ends + cost of sides

eliminate h

$C(r) =$ ______

Calculus

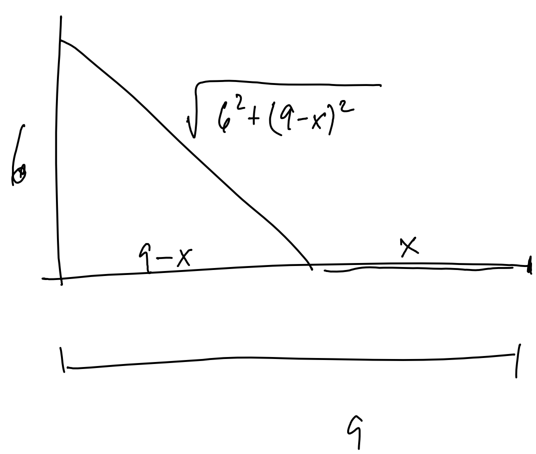


$l \cdot w = 400 \Rightarrow w = \frac{400}{l}$

$C = 2l + w + w$

$= 2l + 2w$

$C(l) = 2l + 2 \cdot \frac{400}{l}$



§5.3 Derivatives of logarithms

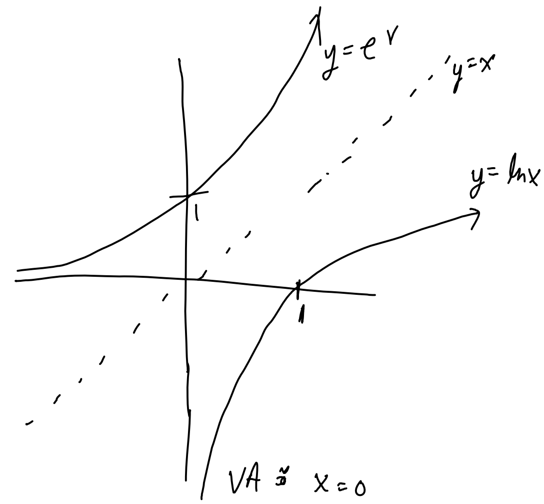
$\frac{d}{dx} e^x = e^x$, $\frac{d}{dx} e^{f(x)} = e^{f(x)} f'(x)$

$y = \ln x \iff e^y = x$ $(y = \log_b x \iff b^y = x)$

$\frac{d}{dx} e^y = \frac{d}{dx} x$

$e^y y' = 1$

$y' = \frac{1}{e^y} = \frac{1}{x}$



$\frac{d}{dx} \ln x = \frac{1}{x}$

$\frac{d}{dx} \ln(f(x)) = \frac{1}{f(x)} f'(x) = \frac{f'(x)}{f(x)}$

ex. $y = \ln(x^2+5) \Rightarrow y' = \frac{1}{x^2+5} 2x = \frac{2x}{x^2+5}$

ex. #20/pg 333 $f(x) = \ln(x^4+5)^{3/2} = \frac{3}{2}\ln(x^4+5)$ $\log_b M^N = N \log_b M$

$f'(x) = \frac{3}{2} \cdot \frac{1}{x^4+5} \cdot 4x^3 = \frac{6x^3}{x^4+5}$

#22/pg 333 $f(x) = [\ln(x^4+5)]^{3/2}$

$f'(x) = \frac{3}{2}[\ln(x^4+5)]^{1/2} \frac{1}{x^4+5} 4x^3$

#30 $f(x) = \frac{\ln x}{e^x+1} \rightarrow f'(x) = \frac{(e^x+1)\cdot\frac{1}{x} - (\ln x)(e^x)}{(e^x+1)^2}$

Other bases

$y = b^x$

$\ln y = \ln b^x$

$\ln y = x \ln b$

$e^{\ln y} = e^{x \ln b}$

$y = e^{x \ln b}$

$y' = e^{x \ln b} \ln b$

$y' = b^x \ln b$

$\frac{d}{dx} b^x = b^x \ln b$

e.g. $y = 2^x$ - return to base e

$y = 2^x$

$\ln y = \ln 2^x$

$\ln y = x \ln 2$

$e^{\ln y} = e^{x \ln 2}$

$y = e^{x \cdot \ln 2}$

$y' = e^{x \ln 2} \cdot \ln 2$

$y' = 2^x \ln 2$

$\frac{d}{dx} 3^x = 3^x \ln 3$, $\frac{d}{dx} 5^{x^2+1} = 5^{x^2+1} \ln 5 \, (2x)$

logs

$y = \log_2 x \Rightarrow 2^y = 2^{\log_2 x}$

$2^y = x$

$\ln 2^y = \ln x$

$y \ln 2 = \ln x$

$y = \frac{1}{\ln 2} \ln x$

$\hookrightarrow y' = \frac{1}{\ln 2} \cdot \frac{1}{x}$

$\log_2 x = \frac{1}{\ln 2} \ln x$

$y = \log_b x \iff b^y = b^{\log_b x} \Rightarrow b^y = x$

$$\ln b^y = \ln x$$
$$y \ln b = \ln x$$
$$y = \frac{1}{\ln b} \ln x$$
$$y' = \frac{1}{\ln b} \frac{1}{x}$$

$$\frac{d}{dx} \log_b x = \frac{1}{\ln b} \cdot \frac{1}{x}$$
$$\frac{d}{dx} b^x = \ln b \cdot b^x$$

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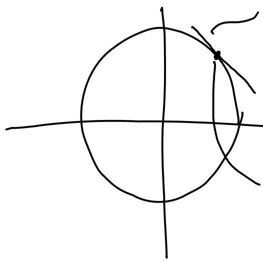
§5.5 Implicit Differentiation (Chain Rule + Bookkeeping)



explicit differentiation to find slope

solve for y & ...

$$y^2 = 1 - x^2$$
$$y = \pm\sqrt{1-x^2}$$
$$y = +\sqrt{1-x^2} \quad \text{(Top half)}$$

$$y' = \frac{1}{\cancel{2}}(1-x^2)^{-1/2}(-\cancel{2}x) = \frac{-x}{\sqrt{1-x^2}}$$

evaluate at $x = 1/\sqrt{2}$

$$y' = \frac{-1/\sqrt{2}}{\sqrt{1-(1/\sqrt{2})^2}} = \frac{-1/\sqrt{2}}{\sqrt{1-1/2}} = \frac{-1/\sqrt{2}}{\sqrt{1/2}} = \frac{-1/\sqrt{2}}{1/\sqrt{2}} = -1$$

implicit differentiation think of y as a function of x & differentiate

→ i.e. "$y = f(x)$"

$$x^2 + (f(x))^2 = 1 \qquad \text{want } f'(x)$$

$$\frac{d}{dx}x^2 + \frac{d}{dx}(f(x))^2 = \frac{d}{dx}1$$

$$2x + 2f(x)\underline{f'(x)} = 0$$

$$2f(x)f'(x) = -2x$$

$$f'(x) = \frac{-\cancel{2}x}{\cancel{2}f(x)} = \frac{-x}{y} = \frac{-1/\sqrt{2}}{1/\sqrt{2}} = -1 \quad \text{!!}$$

$x^2 + y^2 = 1$ find y'

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}1$$

$$2x + 2yy' = 0$$

$$y' = \frac{-2x}{2y} = \frac{-x}{y} = \frac{-1/\sqrt{2}}{1/\sqrt{2}} = -1$$

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$$(f(x))^2 - f(x) - 4x = 0$$

y' at $(0,1)$ $\rightarrow$ $y^2 - y - 4x = 0$

$2f(x)f'(x) - f'(x) - 4 = 0$

$$2yy' - y' - 4 = 0$$
$$2yy' - y' = 4$$
$$(2y-1)y' = 4$$
$$y' = \frac{4}{2y-1}$$
$$y'\Big|_{(0,1)} = \frac{4}{2(1)-1} = \frac{4}{1} = 4$$

#30 tangent line(s) w/ $x = 1$ to $xy^2 - y - 2 = 0$

$x = 1$ $\rightarrow$ $y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

2 points $y = 2$ $y = -1$

$(1,2)$ or $(1,-1)$

to get slope we need $\frac{dy}{dx}$ or y'

$$\frac{d}{dx}\left(xy^2 - y - 2\right) = \frac{d}{dx}0$$

$$\underset{F}{x}\ \underset{S'}{2yy'} + \underset{F'}{1}\ \underset{S}{y^2} - y' - \cancel{0} = 0$$

$$2xyy' - y' = -y^2$$
$$(2xy-1)y' = -y^2$$
$$y' = \frac{-y^2}{2xy-1}$$

...