59/4.3 $f(x) = x + \frac{1}{x} + \frac{4}{x^3} = \frac{x^4+x^2+4}{x^3}$

Analyze f: domain : $x \neq 0$,

VA : $x = 0$

oblique asymp. : $y = x$

HA : NA

intercepts: y-int : none ($f(0)$ undef.)

x-int : ?

$$x + \frac{1}{x} + \frac{4}{x^3} = 0$$

$$\frac{x^3}{x^3} x + \frac{x^2}{x^2} \cdot \frac{1}{x} + \frac{4}{x^3} = 0$$

$$\frac{x^4+x^2+4}{x^3} = 0 \quad \text{No solution}$$

no x-int.

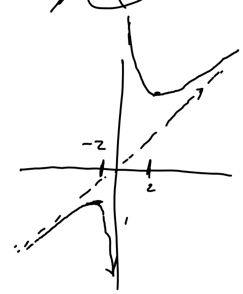
Analyze f'

$$f'(x) = \frac{x^3(4x^3+2x) - (x^4+x^2+4)(3x^2)}{x^6}$$

$$= \frac{\cancel{4x^6} + \cancel{2x^4} - \cancel{3x^6} - \cancel{3x^4} - 12x^2}{x^6} = \frac{x^6 - x^4 - 12x^2}{x^6} = \frac{\cancel{x^2}(x^4-x^2-12)}{\cancel{x^6}\,x^4}$$

$$= \frac{(x^2-4)(x^2+3)}{x^4} = \frac{(x-2)(x+2)(x^2+3)}{x^4}$$

$$\begin{array}{ccccccccc} + + + + + + & 0 & - - & ND & - - & 0 & + + + + + & f' \\ & -2 & & 0 & & 2 & & x \end{array}$$



$\boxed{\begin{array}{l} \text{local max. at } x = -2, \quad y = f(-2) = -2 + \frac{1}{-2} + \frac{4}{(-2)^3} = 5 \\ \text{local min at } x = +2 \quad y = f(2) = 2 + \frac{1}{2} + \frac{4}{2^3} = 5 \end{array}}$

odd function
$f(-x) = -f(x)$
↳ symmetric around origin.

$f(x) = x + \frac{1}{x} - \frac{4}{x^3}$

Analyze f''

start w/ $f'(x) = \frac{x^4-x^2-12}{x^4}$

$$f''(x) = \frac{x^4(4x^3-2x) - (x^4-x^2-12)4x^3}{x^8} = \frac{\cancel{4x^7} - 2x^5 - \cancel{4x^7} + 4x^5 + 48x^3}{x^8}$$

$$= \frac{2x^5+48x^3}{x^8} = \frac{2x^3(x^2+24)}{x^8} = \frac{2(x^2+24)}{x^5}$$

partition # at 0

$$\begin{array}{ccc} - - - - - - - - & ND & + + + + + + + + \quad f'' \end{array}$$

$Q \qquad\qquad\qquad x$

ex. $f(x) = (20x^2+9)^4$

$f'(x) = 4(20x^2+9)^3\,40x = \underbrace{160x\,(20x^2+9)^3}_{PR}$

$$f''(x) = \overset{F}{160x}\ \overset{D\,2^{nd}}{3(20x^2+9)^2(40x)} + \overset{D\,1^{st}}{160}\ \overset{2^{nd}}{(20x^2+9)^3}$$

(PR)

(CR)

$= 160\,(20x^2+9)^2\left[120x^2 + 20x^2 + 9\right]$

§4.5 Optimization!

(0) Two Words

maximize or minimize — Quantity

(1) draw picture (if poss.)
- & label
  OR introduce variables
- find relationships (eqn's)
- get one function for Q.

(2) find & JUSTIFY max/min

methods

(a) non-calculus — use structure of Q, e.g. quadratic (go to vertex)

(b) domain of Q is closed interval, then use Closed Interval Method (critical values of Q ($Q' = 0$ or Q' DNE) + endpoints)

(c) domain of Q is open (infinite), then use a sign chart of Q' (inc/dec) to justify max/min

(3) Make S...

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find 2 pos. #'s whose ~~product~~ sum is 60 and whose product is maximum and what product is maximum.

(3) make sure you've answered the question.

(0) maximize product

(1) x, y the two numbers

$$\boxed{\begin{matrix} x+y=60 \\ Q=xy \end{matrix}} \begin{matrix} \to y=60-x \\ \to Q=x(60-x) \end{matrix}$$

$Q = 60x - x^2$ ← quadratic

(2) max. of downward parabola occurs at vertex at $x=30$ (vtx. halfway b/w zeroes)

(3) the two #'s are $\underline{30}$ & $60-30=\underline{30}$

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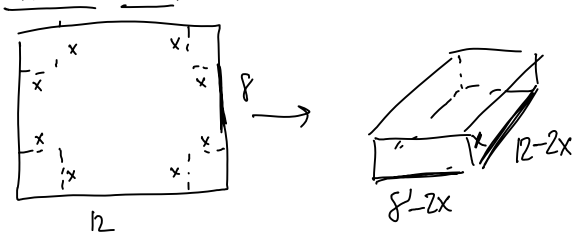
(0) maximize volume

(1)



$V(x) = x(8-2x)(12-2x)$

cubic (foil)

$x \in [0,4]$

Closed Interval Method