More 4.2, 4.3

Monday, June 29, 2009
7:11 AM

§4.2 before exam → concavity & inflection points

$x = 3$
$y = ?$
$y = f(3)$

$- \quad\quad + \quad f''$ (sign chart, 3 marked)

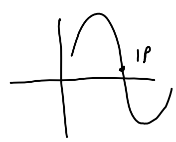
For max's, min's, Inflection points always give x & y - coordinates!
- x coordinate from sign chart
- y coordinate from $f(x)$

Strategy for graphing/understanding functions

Step 1 Analyze f:
- find intercepts (y-int is easy)
  x-ints may be hard (estimate or don't find)
- domain of f (what x's are valid)
- vertical asymptotes (divide by 0 — "$\frac{\neq}{0}$")
  (a hole could be "$\frac{0}{0}$")
- horizontal asymptote $\lim_{x\to\infty} f(x)$ (rescale — divide through by largest term)

Step 2 Analyze f'
- sign chart for f'
- increasing/decreasing intervals
- local extrema (don't forget y-coordinates)

Step 3 Analyze f''
- sign chart for f''
- CU / CD
- inflection points (y-coord's!)
  are/can also be points of max/min steepness



Step 4 put it all together → graph

§4.2 style ex. $f(x) = \frac{1}{4}x^4 + 2x^3 + \frac{9}{2}x^2 - 1$

Step 1 Domain: $(-\infty, \infty)$
y-int: -1
x-int's: hard

Step 2 $f'(x) = x^3 + 6x^2 + 9x = x(x^2+6x+9) = x(x+3)^2$

$- - 0 - - - 0 + + + + +$ (sign chart: -3, 0, x)

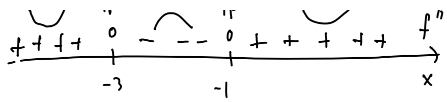
f decreases $(-\infty, 0)$
f increases on $(0, \infty)$

local (absolute) min at $x=0$, $y = f(0) = -1$

horizontal tangent at $x = -3$, $y = f(-3) = \frac{1}{4}(-3)^4 + 2(-3)^3 + \frac{9}{2}(-3)^2 - 1$
$= \frac{81}{4} - 54 + \frac{81}{2} - 1 = 5.75$

Step 3 $f''(x) = 3x^2 + 12x + 9 = 3(x^2 + 4x + 3)$
$= 3(x+3)(x+1)$

$y = 5.75$ IP $\quad\quad$ $y =$ IP

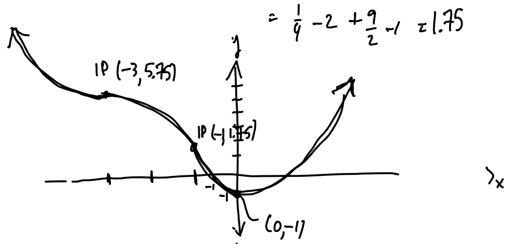


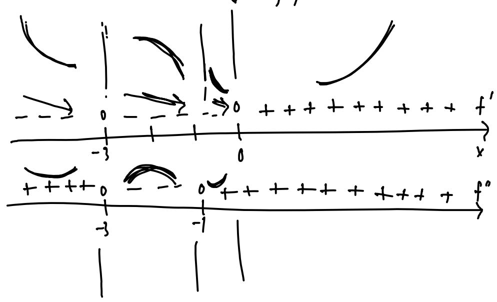
CU on $(-\infty,-3)\cup(-1,\infty)$, CD $(-3,-1)$

IP $x=-3$, $y=+5.75$

$x=-1$, $y=f(-1)=\frac{1}{4}(-1)^4+2(-1)^3+\frac{9}{2}(-1)^2-1$

$=\frac{1}{4}-2+\frac{9}{2}-1=1.75$





§4.3 Rational Functions — same strategy – now w/ asymptotes & occasional holes (oblique asymptote)

#42 / pg 272 $\quad f(x)=\dfrac{2x^2+5}{4-x^2}$

Step 1 Domain: $x\neq\pm2$.

VA's: (for $x\approx L$, $f(x)\approx\frac{B}{\text{small}}$), VA's at $x=\pm2$.

Int's y-int: $y=f(0)=\frac{5}{4}=1.25$

x-ints $0=f(x)$

$$\frac{2x^2+5}{4-x^2}=0$$

$2x^2+5=0$ — no solutions

$2x^2=-5$

$x^2=-5/2$ → f doesn't intersect x-axis.

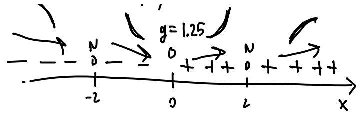
~~$x=\pm\sqrt{2.5}$~~

H.A. $\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to\infty}\dfrac{(2x^2+5)\frac{1}{x^2}}{(4-x^2)\frac{1}{x^2}}=\lim\limits_{x\to\infty}\dfrac{2+5/x^2}{4/x^2-1}$

$=\dfrac{2+0}{0-1}=\dfrac{2}{-1}=-2$ so $y=-2$ is a HA

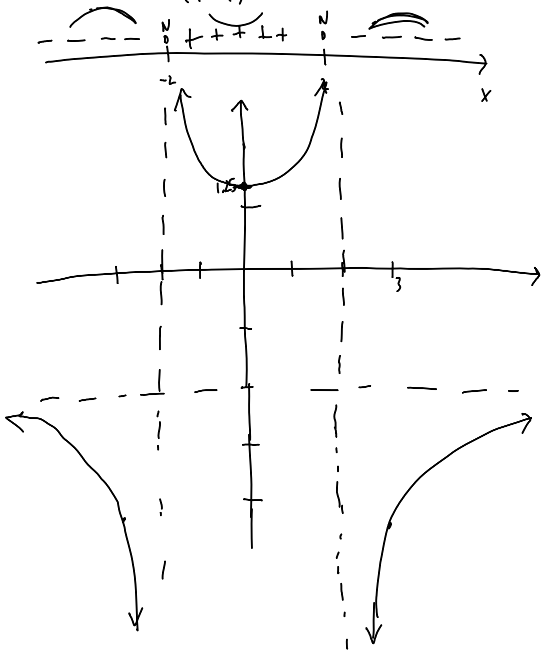
Analyze f': $f(x)=\dfrac{2x^2+5}{4-x^2}$

$$f'(x)=\frac{(4-x^2)(4x)-(2x^2+5)(-2x)}{(4-x^2)^2}=\frac{16x-\cancel{4x^3}+\cancel{4x^3}+10x}{(4-x^2)^2}=\frac{26x}{(4-x^2)^2}$$



Analyze $f''(x) = \dfrac{(4-x^2)^2\, 26 - 26x\, 2(4-x^2)^1(-2x)}{(4-x^2)^4} = \dfrac{\cancel{(4-x^2)}\, 26\,[4-x^2+4x^2]}{(4-x^2)^{\cancel{4}\,3}}$

$= \dfrac{(3x^2+4)\,26}{(4-x^2)^3}$



in wide open intervals plot another point

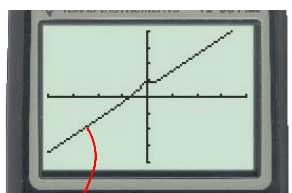
$y = f(3)$

$= \dfrac{2(3)^2+5}{4-3^2}$

$= \dfrac{18+5}{4-9} = \dfrac{23}{-5}$

$= -4.6$

$f(x) = \dfrac{x^3+3x^2+5}{x^2+1}$



oblique asymptotes occur when degree of numerator is one higher than denominator

what is the equation of this line?

to find equation $\rightarrow$ Long Division.

$$\begin{array}{r} x + 3 \\ x^2+1 \,\overline{)\, x^3 + 3x^2 + 5} \\ \underline{x^3 + x} \\ 3x^2 - x + 5 \\ \underline{3x^2 + 3} \\ -x + 2 \end{array}$$

~~x^2+1~~ ~~d~~

remainder is $-x+2$

... to zero for large x

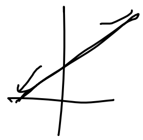
goes to 0

So $$\frac{x^3+3x^2+5}{x^2+1} = x+3 + \frac{-x+2}{x^2+1}$$

line

plus a part that gets small for large values of x

oblique asymptote



$$\boxed{f(x) = x - \frac{16}{x^3}}$$