4.1-4.2

Tuesday, June 23, 2009
7:26 AM

§4.1 How derivatives affect the shape of a graph

Analyze sign chart of f'	$\longrightarrow$	shape of f
$f' > 0$	$\longrightarrow$	$f \nearrow$
$f' < 0$	$\longrightarrow$	$f \searrow$
$f' = 0$	$\longrightarrow$	f has a horizontal tangent
$f'(x)$ undefined while $f(x)$ is defined	$\longrightarrow$	f has a cusp or vertical tangent

Any place where $f'(x)$ can change sign is called a partition number.

The critical values of f are values of x that are in the domain of f (so $f(x)$ is defined) & $f'(x) = 0$ or $f'(x)$ undefined.

(a critical value is a partition # where $f(x)$ is defined)

Ex. $f(x) = \frac{1}{x-3}$, $f'(x) = \frac{-1}{(x-3)^2}$

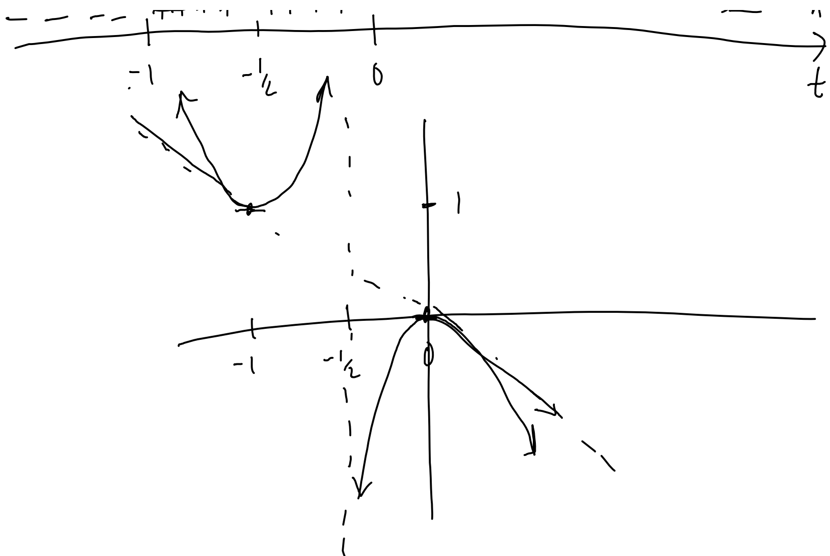
$x = 3$ is a partion # but not a critical value ($f(3)$ is undefined)

Ex. $h(t) = \frac{-t^2}{2t+1}$, $h'(t) = \frac{-2t(t+1)}{(2t+1)^2}$ (from Quiz 4)

partition #'s of h': $t = -\frac{1}{2}, 0, -1$ (places where

critical values of h: $t = 0, -1$ (h' is 0 or undefined & h is defined)

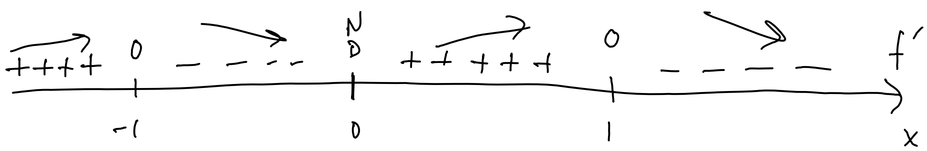
0 + + + + ND + + + + + 0 – – – – – – – – h'

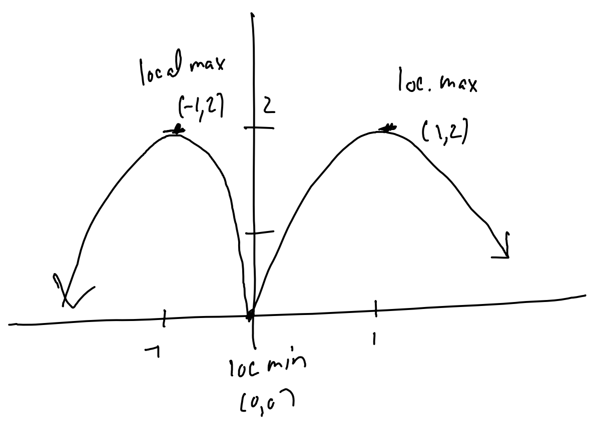


54/p240 $f(-1)=2$, $f(0)=0$, $f(1)=2$, $f'(-1)=0$, $f'(1)=0$, $f'(0)$ undefined

$f'(x)>0$ on $(-\infty,-1)$ and $(0,1)$

$f'(x)<0$ on $(-1,0)$ and $(1,\infty)$





#90 pg 243 $C(x)=450+30x+.08x^2$ $\quad 0\le x\le 200$, x blenders

(A) $\bar{C}(x)=\dfrac{C(x)}{x}=\dfrac{450+30x+.08x^2}{x}$

$$=\frac{450}{x}+\frac{30x}{x}+\frac{.08x^2}{x}$$

$$=\frac{450}{x}+30+.08x \quad = 450x^{-1}+30+.08x$$

(b) $\bar{C}'(x) = 450(-1)x^{-2} + 0 + .08 = \frac{-450}{x^2} + .08$

$= \frac{-450}{x^2} + \frac{.08x^2}{x^2} = \frac{.08x^2 - 450}{x^2} \rightarrow$ partition #'s

$\bar{C}'(x) = 0 \rightarrow \mathbb{R}$

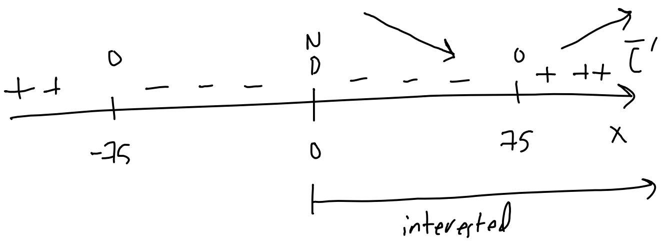
$.08x^2 - 450 = 0$

$x^2 = 450/.08 = 5625$

$x = \pm 75$

OR

$\bar{C}'(x)$ undefined at $x = 0$



average cost has an absolute (also local) minimum at $x = 75$ blenders

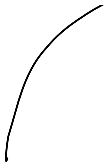
it decreases on $(0, 75)$ and increases $(75, \infty)$

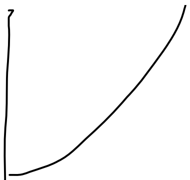
minimum average cost is $\bar{C}(75) = \frac{450}{75} + 30 + .08(75) = \$42$/blender.

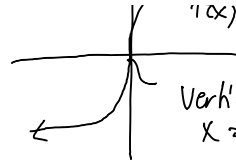
§4.2 2nd Derivatives & the shape of the graph

$f(x) = \sqrt[3]{x} = x^{1/3}$ defined $(-\infty, \infty)$

$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}} = \frac{1}{3(x^2)^{1/3}}$







$'(x) = \frac{1}{3\sqrt[3]{x^2}}$

Vertical tangent line at $x=0$, no slope, no derivative.

$f' > 0$

Slopes are decreasing

derivative of slopes is negative

derivative of f' is neg.

$\frac{d}{dx} f' < 0$

$f' > 0$

Slopes are increasing

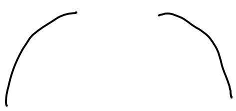
derivative of slopes is positive.

derivative of f' is positive

$\frac{d}{dx} f' > 0$

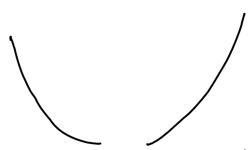
$f'' < 0$

concave down



$f'' > 0$

Concave up



4.2 Level A problems

finish 4.1