3.7 - 4.1

Monday, June 22, 2009
7:13 AM

Lo D hi hi D lo

QR

#72/pg 201

$$\frac{d}{dx}\frac{x^2}{\sqrt{x^2+1}} = \frac{(x^2+1)^{1/2}\,2x - x^2\,\cancel{\tfrac{1}{2}}(x^2+1)^{-1/2}\,\cancel{2}x}{\left((x^2+1)^{1/2}\right)^2}$$

$$= \frac{2x(x^2+1)^{1/2} - x^3(x^2+1)^{-1/2}}{(x^2+1)} = \frac{x(x^2+1)^{-1/2}\left[2(x^2+1)^1 - x^2\right]}{x^2+1}$$

$$= \frac{x\left[2x^2+2-x^2\right]}{(x^2+1)^{1/2}(x^2+1)^1} = \frac{x(x^2+2)}{(x^2+1)^{3/2}}$$

#14/pg 211 $x = 9000 - 30p$ $C(x) = 150{,}000 + 30x$ x T.V. sets

(A) find p as a function of x
solve for p

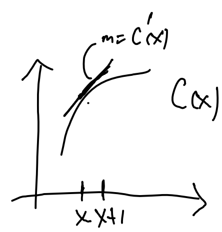
$$30p + x = 9000$$
$$30p = 9000 - x$$
$$p = \frac{1}{30}(9000 - x)$$

$$p = 300 - \frac{1}{30}x$$

$0 \le x \le 9000$
domain

$p = 0$ when
$300 - \frac{1}{30}x = 0$
$300 = \frac{1}{30}x$
$30 \cdot 300 = x$
$x = 9000$

(B) marginal cost $= C'(x)$



exact cost of producing $(x+1)$'st tv set
is $C(x+1) - C(x) \approx C'(x)$

$C'(x) = 30$ \$/TV   (~~about~~ exactly \$30 to produce next TV)

(C) Revenue $R(x) = \text{price} * \text{qty}$

$$R(x) = \left(300 - \frac{1}{30}x\right)x = 300x - \frac{1}{30}x^2$$

$$0 \le x \le 9000$$

(D) marginal revenue

$$R'(x) = 300 - \frac{1}{15}x \quad \$/TV$$

(E) $R'(3000) = 300 - \frac{1}{15}(3000) = 300 - 200 = \$100/TV$

when producing 3000 TV's revenue is increasing at about \$100 per TV.

or The approximate revenue from the 3001'st TV is \$100.

$R'(6000) = 300 - \frac{1}{15}(6000) = -\$100/TV$

when producing 6000 TV's total revenue is decreasing at about \$100 per TV

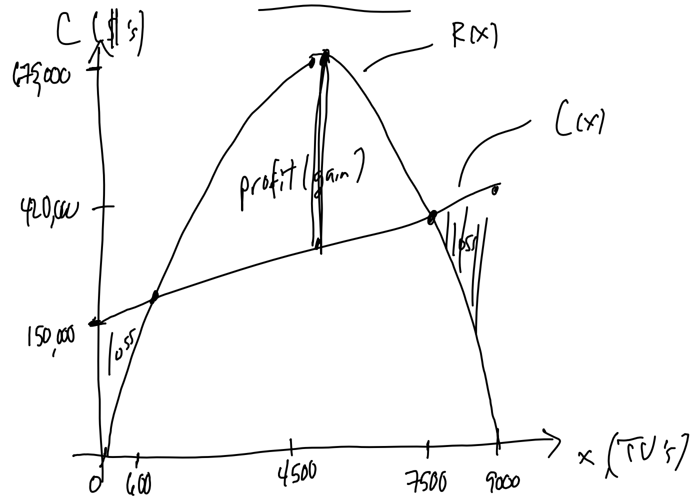
OR

The approx. revenue from the ~~300~~ 6001 st TV is $-\$100$.

(F) $0 \leq x \leq 9000$

$C(x) = 150{,}000 + 30x$

$R(x) = (300 - \frac{1}{30}x)x = 300x - \frac{1}{30}x^2$



find break even points (x-values)

$$R(x) = C(x)$$

$$\left(300 - \frac{1}{30}x\right)x = 150000 + 30x$$

$$300x - \frac{1}{30}x^2 - 150000 - 30x = 0$$

$$-\frac{1}{30}x^2 + 270x - 150000 = 0$$

$$x = \frac{-270 \pm \sqrt{270^2 - 4(-\frac{1}{30})(-15000)}}{2(-\frac{1}{30})} = \$600 \text{ or } 7500$$

(I) Profit $\quad P(x) = R(x) - C(x)$

(H) marginal profit $\quad P'(x)$

(J) interpret $\quad P'(1500) = \$72/\text{TV}$ (made up) profit is increasing by approx. \$72. for each added TV when 1500 TV's are produced.

OR

the approx. profit from the 1501st TV is \$72

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average cost $= \dfrac{\text{total cost}}{\text{\# of items produced}} = \dfrac{C(x)}{x} = \overline{C}(x)$

average revenue $\quad \overline{R}(x) = \dfrac{R(x)}{x}$

average profit $\quad \overline{P}(x) = \dfrac{P(x)}{x}$

e.g. $\quad R(x) = (300 - \frac{1}{30}x)\, x = 300x - \frac{1}{30}x^2 \quad$ (Total revenue in \$ from x TV's)

average revenue

$$\overline{R}(x) = \frac{R(x)}{x} = \frac{300x - \frac{1}{30}x^2}{x} = 300 - \frac{1}{30}x$$

HW 3.7

Now §4.1 | Relationship between f' & f

Theorem 1

if $f' > 0$ then f is increasing

if $f' < 0$ then f is decreasing

$\boxed{\#28}$ $\quad f(x) = -2x^3 + 3x^2 + 120x$

$$\begin{aligned} f'(x) &= -6x^2 + 6x + 120 \\ &= -6(x^2 - x - 20) \\ &= -6(x-5)(x+4) \end{aligned}$$

x		-4		5	
f'	$- - -$	0	$+ + + + +$	0	$- - -$

→ f is increasing on $(-4, 5)$

f is decreasing on $(-\infty, -4]$ and $(5, \infty)$

at $x = -4$ f has a local minimum (because sign chart shows

$f(-4) = -2(-4)^3 + 3(-4)^2 + 120(-4)$

$= -304$

− − − − + + + +

−4

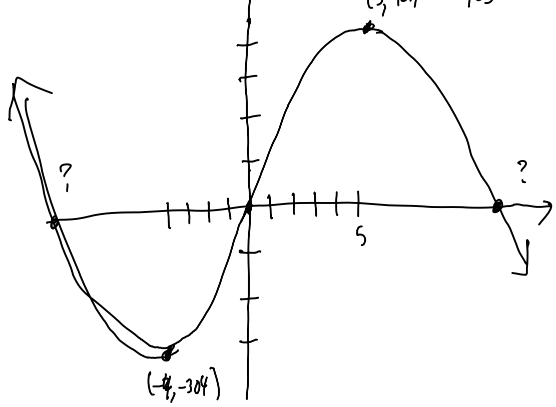
at $x = 5$ f has a local maximum

$f(5) = -2(5)^3 + 3(5)^2 + 120(5)$

(+ + + + − − − f′)

5

$(5, 425) = 425$



Start 4.1