3.4-3.5

Wednesday, June 17, 2009
7:12 AM

Derivative Notation: $y = f(x) \rightarrow$ derivative $f'(x) \equiv \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

other ways to write: $y', f', \frac{dy}{dx}, \frac{df}{dx}, \frac{d}{dx}f$ (be able to use for poly, simple rational, or root)

Derivative Shortcuts:

$\frac{d}{dx}c = 0$, $\frac{d}{dx}x = 1$, $\frac{d}{dx}x^2 = 2x$, $\frac{d}{dx}x^3 = 3x^2$

$$\boxed{\text{Power Rule: } \frac{d}{dx}x^n = nx^{n-1}, \quad n \text{ any real}}$$

Ex $f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2}\cdot\frac{1}{x^{1/2}} = \frac{1}{2\sqrt{x}}$

Ex $g(t) = t^{-3}$

$g'(t) = -3t^{-4}$

$h(t) = \frac{1}{t^5} = t^{-5}$

$h'(t) = -5t^{-6} = \frac{-5}{t^6}$

Sums & Diff's & Constants

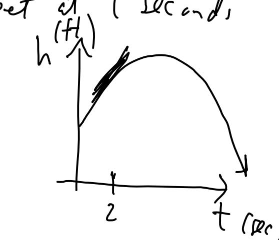
$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$ (take the derivative of each piece in a sum/diff. separately)

$\frac{d}{dx}cf(x) = c\frac{d}{dx}f(x)$ (multiplicative constants are along for the ride)

Ex. $r(t) = 540 + 400t - 32t^2$ ← rocket height in feet at t seconds

$r'(t) = 0 + 400 - 32\cdot 2t = 400 - 64t$ ft/s

↑ instantaneous rate of change in height at time t seconds



at $t=2$ the velocity of the rocket is $r'(2) = 400 - 64(2) = 272 \text{ ft/sec}$

#38 / pg 183 $\quad \frac{d}{dx}\left(\frac{5x^3}{4} - \frac{2}{5x^3}\right) = \frac{d}{dx}\left(\frac{5}{4}x^3 - \frac{2}{5}x^{-3}\right)$

$$= \frac{5}{4}\cdot 3x^2 - \frac{2}{5}(-3)x^{-4} = \frac{15}{4}x^2 + \frac{6}{5x^4}$$

#46 find w' if $w = \frac{10}{\sqrt[5]{u}} = \frac{10}{u^{1/5}} = 10u^{-1/5}$

$$w' = 10\left(-\frac{1}{5}\right)u^{-1/5-1} = 10\left(\frac{-1}{5}\right)u^{-6/5} = -2u^{-6/5}$$

fine

$$= \frac{-2}{u^{6/5}} = \frac{-2}{(u^6)^{1/5}} = \frac{-2}{\sqrt[5]{u^6}}$$

fine

#74 if $y = \frac{5x-3}{15x^6} = \frac{5x}{15x^6} - \frac{3}{15x^6} = \frac{1}{3x^5} - \frac{1}{5x^6} = \frac{1}{3}x^{-5} - \frac{1}{5}x^{-6}$

$$y' = \frac{1}{3}(-5)x^{-6} - \frac{1}{5}(-6)x^{-7}$$

$$= \frac{-5}{3x^6}\cdot\frac{x5}{x5} + \frac{6}{5x^7}\frac{3}{3} = \frac{-25x}{15x^7} + \frac{18}{15x^7} = \frac{-25x+18}{15x^7}$$

§3.5 Product & Quotient Rule

pg 187 $\quad y = f(x) = F(x)S(x)$ $\quad \boxed{\text{Product Rule}}$ $\quad \cancel{y' \neq F'S'}$

$$y' = f'(x) = F(x)S'(x) + F'(x)S(x)$$

Ex. $y = \underbrace{(3x^5)}_{F}\underbrace{(5x^7 - 9x^2)}_{S}$

$$y' = \underset{F}{(3x^5)}\,\underset{S'}{(35x^6 - 18x)} + \underset{F'}{(15x^4)}\,\underset{S}{(5x^7 - 9x^2)}$$

$$= 105x^{11} - 54x^6 + 75x^{11} - 135x^6$$

$$= 180x^{11} - 189x^6$$

Again w/o product rule $y = 15x^{12} - 27x^7$

$$y' = 180x^{11} - 189x^6$$

Quotient Rule $y = f(x) = \dfrac{hi}{lo} = \dfrac{lo\ D\ hi - hi\ D\ lo}{lo^2}$

"lo D hi minus hi D lo over lo squared"

$$y = f(x) = \frac{T(x)}{B(x)} = \frac{B(x)\,T'(x) - T(x)\,B'(x)}{(B(x))^2}$$

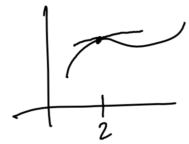
18/pg 192 $f(x) = \dfrac{x^2 - 4}{x^2 + 5}$

$$f'(x) = \frac{\overset{lo}{(x^2+5)}\,\overset{D\ hi}{(2x)} - \overset{hi}{(x^2-4)}\,\overset{D\ lo}{(2x)}}{\underset{lo^2}{(x^2+5)^2}}$$

$$= \frac{\cancel{2x^3} + 10x - \cancel{2x^3} + 8x}{(x^2+5)^2} = \frac{18x}{(x^2+5)^2}$$

#38/pg 192 $f(x) = 2x - 5$ find eq'n of line tangent to graph at $x = 2$

$2x-3$



need to find $f'(x)$, then evaluate at $x=2$ to get m

$$f'(x) = \frac{\overset{lo}{(2x-3)}\overset{D\,hi}{(2)} - \overset{hi}{(2x-5)}\overset{D\,lo}{(2)}}{(2x-3)^2}$$

$$y - y_0 = m(x - x_0)$$

$y_0 = f(2) = \frac{2(2)-5}{2(2)-3} = \frac{-1}{1} = -1$

$x_0 = 2$

$m = f'(2) = 4$

$$m = f'(2) = \frac{(2(2)-3)(2) - (2(2)-5)(2)}{(2(2)-3)^2} = \frac{2 - (-1)(2)}{1^2} = 4$$

$$y - (-1) = 4(x-2)$$

$$y + 1 = 4x - 8$$

$$\boxed{y = 4x - 9}$$