3.3-3.4

Tuesday, June 16, 2009
7:12 AM

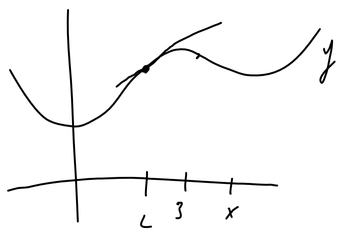
the derivative of $f(x)$ is denoted by $f'(x)$ and is given by

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

poly's

rationals

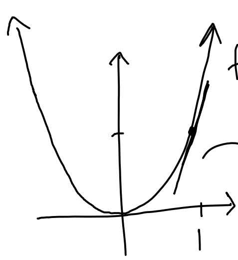
roots (square)



ex. $f(x) = x^2$, $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} = \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x$$

$f'(x) = 2x \leftarrow$ $\boxed{\text{Slope Function}}$



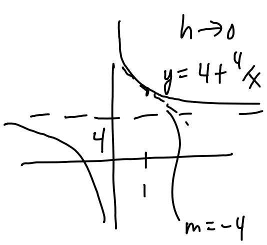
slope of the tangent line is $f'(1) = 2(1) = 2$.

#19 on pg 173 $f(x) = 4 + 4/x$, find $f'(x)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\left(\cancel{4} + \frac{4}{x+h}\right) - \left(\cancel{4} + \frac{4}{x}\right)}{h} = \lim_{h\to 0} \frac{\frac{4}{x+h} - \frac{4}{x}}{h}$$

$$= \lim_{h\to 0} \left(\frac{4}{(x+h)}\frac{x}{x} - \frac{4}{x}\frac{(x+h)}{(x+h)}\right)\cdot\frac{1}{h} = \lim_{h\to 0} \left(\frac{4x}{(x+h)x} - \frac{4x+4h}{x(x+h)}\right)\frac{1}{h}$$

$$= \lim_{h\to 0} \left(\frac{-4\cancel{h}}{x(x+h)}\right)\frac{1}{\cancel{h}} = \lim_{h\to 0} \frac{-4}{x(x+h)} = \frac{-4}{x\cdot x} = \frac{-4}{x^2}$$



at $x=1$, the slope of the tangent line is

$$f'(1) = \frac{-4}{1^2} = -4$$

ex. $f(x) = \sqrt{x+2}$

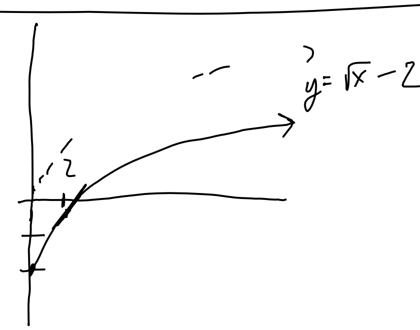
$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h} \cdot \frac{(\sqrt{x+h+2} + \sqrt{x+2})}{(\sqrt{x+h+2} + \sqrt{x+2})}$$

$$= \lim_{h\to 0} \frac{(\cancel{x}+h+\cancel{2}) - (\cancel{x}+\cancel{2})}{h(\sqrt{x+h+2} + \sqrt{x+2})} = \lim_{h\to 0} \frac{\cancel{h}\ 1}{\cancel{h}(\sqrt{x+h+2} + \sqrt{x+2})}$$

$$= \frac{1}{\sqrt{x+2} + \sqrt{x+2}} = \frac{1}{2\sqrt{x+2}}$$

#59/ p 146 ~~pg 146~~ $f(x) = \sqrt{x} - 2$

find slope of the tangent line or the instantaneous rate of change at $x=2$. (at $a=2$)



$$m = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h\to 0} \frac{(\sqrt{2+h} - \cancel{2}) - (\sqrt{2} - \cancel{2})}{h} = \lim_{h\to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

$$= \lim_{h\to 0} \frac{(\sqrt{2+h} - \sqrt{2})(\sqrt{2+h} + \sqrt{2})}{h(\sqrt{2+h} + \sqrt{2})} = \lim_{h\to 0} \frac{\cancel{2}+\cancel{h}-\cancel{2}\ 1}{\cancel{h}(\sqrt{2+h} + \sqrt{2})} = \frac{1}{\sqrt{2} + \sqrt{2}} = \frac{1}{2\sqrt{2}}$$

---

§3.4 Differentiation Rules (shortcuts). $f(x) = x^2 \rightarrow f'(x) = 2x$

if $f(x) = c$ (constant function)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{c - c}{h} = \lim_{h\to 0} \frac{0}{h} = \lim_{h\to 0} 0 = 0$$

if $f(x) = x$

$$f'(x) = \lim_{h\to 0} \frac{(\cancel{x}+h) - \cancel{x}}{h} = \lim_{h\to 0} \frac{h}{h} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}} 1 = 1$$

$f(x) = x^2 \rightarrow f'(x) = 2x$

$f(x) = x^3 \rightarrow$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}} = 3x^2$$

$f(x) = x^n \longrightarrow f'(x) = nx^{n-1}$

$p(x) = 5x^3 - 7x^2 + 13x + 2$

$p'(x) = 5 \cdot 3x^2 - 7 \cdot 2x^1 + 13$

$p'(x) = 15x^2 - 14x + 13$