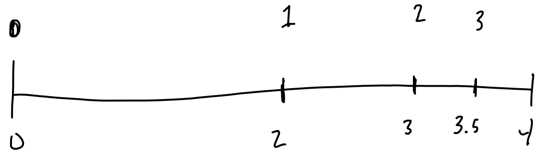
3.1 Limits, 3.2

Thursday, June 11, 2009
7:12 AM

Magic Frog on 4 meter diving board

jumps $2m + 1m + \frac{1}{2}m + \dots$



frog always jumps $\frac{1}{2}$ way to the end !

does the frog get wet ?
if not, how far does the frog jump in total?

Single # for total distance

3.999999

$3.\overline{9}$

4

these are the same # !

total distance frog travels is closer to 4 than to any other number, we could say the distance limits on 4

how many x intercepts does a line w/ non zero slope have ?

$mx + b = 0$

has 1 sol'n

$x = 3.\overline{9}$

$10x = 39.\overline{9}$

$10x - x = 39.\overline{9} - 3.\overline{9}$

$9x = 36$

$x = 4$

Same #

---

what values does f(x) approach as x approaches a — what is the intended or limiting value of f(x) as x gets close to a (as close as the frog did to the end of the board)

we write $\lim_{x \to a} f(x) = L$

this means f(x) gets close to L (like the frog gets close to 4) as x gets close to a (but $x \neq a$)

$\lim_{x \to 2} (x+3) = 5$ | $\lim_{x \to -1} x = -1$ |

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} = 4 \quad \checkmark$$

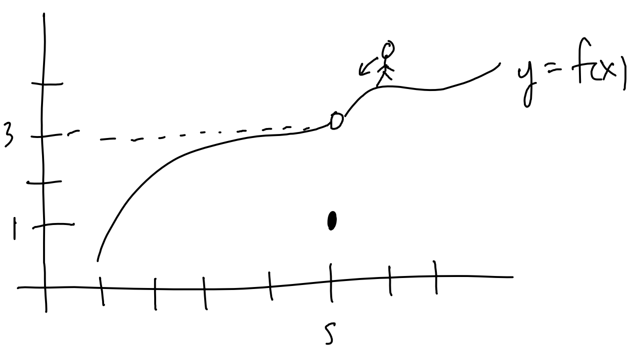
" $0/0$ " is an indeterminate form

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

difference quotient → always gives indeterminate form

$$\lim_{x \to \infty} \frac{1}{x} = 0 \qquad \left(y = 0 \text{ is a H.A. of } f(x) = \frac{1}{x} \right)$$

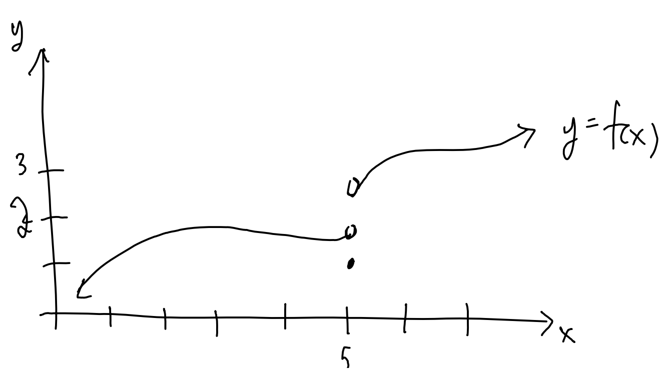
$$\lim_{x \to -32} 6 = 6$$



$$\lim_{x \to 5} f(x) = 3$$

$f(5) = 1$

nothing to do w/ limit as $x \to 5$



$\cancel{\lim_{x\to 5} f(x) =}$

Left limit

$$\lim_{x\to 5^-} f(x) = 2$$

$(x<5)$

Right limit

$$\lim_{x\to 5^+} f(x) = 3$$

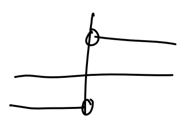
$(x>5)$

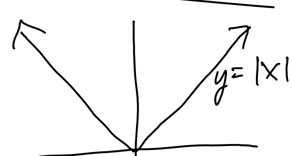
but $\lim_{x\to 5} f(x)$ does not exist (DNE)

Theorem $\lim_{x\to a} f(x) = L$ if and only if $\lim_{x\to a^-} f(x) = L$ $\&$ $\lim_{x\to a^+} f(x) = L$

(to have a limit, must have a left $\&$ right limit that match)

Ex. $\lim_{x\to 0} \frac{|x|}{x}$





$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x<0 \end{cases}$$

Right limit

$$\lim_{x\to 0^+} \frac{|x|}{x} = \lim_{x\to 0^+} \frac{x}{x} = \lim_{x\to 0^+} 1 = 1$$

Left limit

$$\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{x\to 0^-} \frac{-x}{x} = \lim_{x\to 0^-} -1 = -1$$

left $\&$ right limits do not match so $\lim_{x\to 0} \frac{|x|}{x}$ DNE

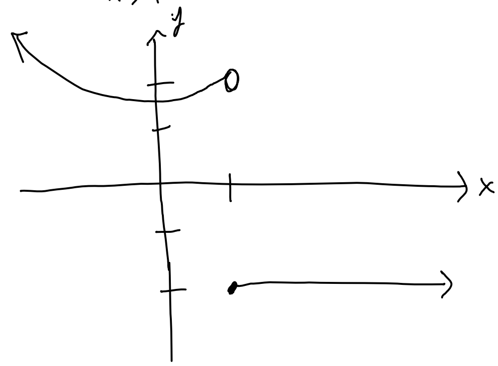
Ex. $f(x) = \sqrt{x}$, $\lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} \cdot \frac{(\sqrt{2+h} + \sqrt{2})}{(\sqrt{2+h} + \sqrt{2})}$

$$= \lim_{h\to 0} \frac{(2+h)-2}{h\left(\sqrt{2+h}+\sqrt{2}\right)} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{2+h}+\sqrt{2}\right)} = \lim_{h\to 0} \frac{1}{\sqrt{2+h}+\sqrt{2}}$$

$$= \frac{1}{\sqrt{2+0}+\sqrt{2}} = \frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

Pg 145 / 36. Sketch a possible graph

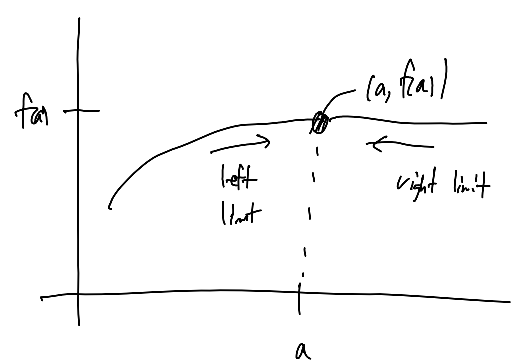
$f(1) = -2$; $\lim_{x\to 1^-} f(x) = 2$; $\lim_{x\to 1^+} f(x) = -2$



§3.2 Continuity

A function $f(x)$ is continuous at $x = a$ if

1. $\lim_{x\to a} f(x)$ exists (need left & right the same)
2. $f(a)$ exists
3. 1 = 2 → the value of the limit is the same as the function value.



polynomials are continuous everywhere, rational functions, exp's, log's are cont. on their domains

Ex.

$$f(x) = \begin{cases} 3x^2+2 & \text{if } x \geq 1 \\ 2x+3 & \text{if } x < 1 \end{cases}$$

is $f(x)$ continuous at $x=1$?

① $\lim_{x\to 1} f(x)$ → Left : $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 2x+3 = 5$

Right : $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 3x^2+2 = 5$

so $\lim_{x\to 1} f(x) = 5$ ✓

② $f(1) = 3(1)^2+2 = 5$ ✓

③ Now we know $\lim_{x\to 1} f(x) = f(1)$ Therefore $f(x)$ is cont. at $x=1$

HW 3.1/3.2