2.2-2.3

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7:18 AM

H.A. $f(x) = \dfrac{x^2+2x-5}{x^3+7x+10}$ $\left(\dfrac{\text{Large}}{\text{Large}}\right)$ rescale

divide by this

$$\to \frac{x^2/x^3 + 2x/x^3 - 5/x^3}{x^3/x^3 + 7x/x^3 + 10/x^3} = \frac{1/x + 2/x^2 - 5/x^3}{1 + 7/x^2 + 10/x^3} \xrightarrow{x \text{ LARGE}}$$

$$= \frac{\text{small}+\text{small}-\text{small}}{1+\text{small}+\text{small}} = \frac{\text{small}}{1} = \text{small} \to 0$$

#39) $2x^3 - x^2 - 7x + 3 = 0$

$x^3 - \frac{1}{2}x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

$|\text{root}| < 1 + \max\left\{\left|-\frac{1}{2}\right|, \left|-\frac{7}{2}\right|, \left|\frac{3}{2}\right|\right\}$

$= 1 + \frac{7}{2} = 9/2 = 4.5$

roots or root is in $(-4.5, 4.5)$

Exponentials (2.2) properties

$a^x a^y = a^{x+y}$ $\quad (3^7 3^8 = 3^{15})$

$(a^x)^y = a^{xy}$ $\quad ((5^3)^6 = 5^{18})$

$\dfrac{a^x}{a^y} = a^{x-y}$ $\quad \left(\dfrac{2^3}{2^5} = 2^{3-5} = 2^{-2} = \dfrac{1}{2^2} = \dfrac{1}{4}\right)$

✓ $a^x = a^y$ if and only if $x = y$

$a^x = b^x$ if and only if $a = b$

for solving equations, eg.

$$2^x = 4^{3x-5}$$
$$2^x = (2^2)^{(3x-5)}$$
$$2^x = 2^{2(3x-5)}$$
$$2^x = 2^{6x-10}$$
$$x = 6x - 10$$
$$5x = 10$$
$$\boxed{x = 2}$$

? $2^2 = 4$

$4^{3x-5} = 4^{3(2)-5} = 4^1 = 4$ ✓

The "natural" base exponential, base e

$$e = 2.71828\,1828 \ldots \quad \text{(nonrepeating = irrational)}$$

Solve $e^{x^2} = e^{2x-5}$

$$x^2 = 2x - 5$$
$$x^2 - 2x + 5 = 0$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(5)}}{2}$$
$$= \frac{2 \pm \sqrt{4 - 20}}{2}$$
$$= \frac{2 \pm \sqrt{-16}}{2} \leftarrow \text{neg.}$$

, no real solutions

§2.3 Logarithms

The logarithm of base b, $b \neq 1$, $b > 0$ is defined by

$$y = \log_b x \quad \text{if and only if} \quad b^y = x$$

$\sqrt{9} = 3$ because $\boxed{3^2 = 9}$

exs

$\log_3 9 = 2$ because $3^2 = 9$

$\log_3 27 = 3$ because $3^3 = 27$

$\log_3 1 = 0$ " $3^0 = 1$

$\log_3 \frac{1}{3} = -1$ " $3^{-1} = \frac{1}{3}$

$\log_2 \frac{1}{8} = -3$ " $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$\log_e e^3 = 3$ " $e^3 = e^3$

base e or natural base

$\ln x = \log_e x$

base 10

$\log_{10} x = \log x$

$\log 100 = 2$ $(10^2 = 100)$

Properties $\log_b (m \cdot n) = \log_b m + \log_b n$

(CAUTION: $\log_b (m+n) \neq \log_b m + \log_b n$)

$\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b b^x = x$, $b^{\log_b x} = x$ inverse properties

$\log_b m^n = n \log_b m$

Ex: solve $e^{2x-5}=7$

$$\ln\left(e^{2x-5}\right)=\ln 7$$
$$2x-5=\ln 7$$
$$2x=5+\ln 7$$
$$x=\frac{5+\ln 7}{2}\approx 3.473$$

solve $\ln(2x)=7$

$$e^{\ln(2x)}=e^7$$
$$2x=e^7$$
$$x=\frac{1}{2}e^7\approx 548.31\ldots$$

Banking Problems:

A = amount at time t

P = principle (present amount at $t=0$)

r = annual rate (decimal)

t = time in years

m = # of compounding periods per year

$$A=P\left(1+\frac{r}{m}\right)^{mt}$$

if m is finite

continuous compounding

$$A=Pe^{rt}$$

if m is infinite

Ex. if we want to buy a CD at $r=.05$ to be worth \$10,000 in 5 years, how much should we pay today? compounded continuously? monthly?

$$\rightarrow P=\frac{A}{e^{rt}}=Ae^{-rt}=10{,}000\,e^{-.05(5)}=10000\,e^{-.25}=\$7788$$

$$P=\frac{A}{\left(1+\frac{r}{m}\right)^{mt}}=\frac{10{,}000}{\left(1+\frac{.05}{12}\right)^{12(5)}}=\$7792.\_\_$$

Ex. How long ~~does~~ does it take to double your \$ if comp. cont. at.

$r=5.5\%$

$A=Pe^{rt}$

what t to get $A=2P$?

$2\cancel{P} = \cancel{P}e^{rt}$

$2 = e^{.055t}$

$\ln 2 = \ln(e^{.055t})$

$\ln 2 = .055t$

$t = \frac{\ln 2}{.055} = 12.6$ years

(rule of thumb for doubling is $\frac{70}{\text{interest rate } \%}$ or $\frac{72}{\text{interest rate } \%}$

Solve $\log_{10} x + \log_{10}(x-4) = \log_{10}(21)$

$\log_{10}(x(x-4)) = \log_{10} 21$

$x(x-4) = 21$

$x^2 - 4x - 21 = 0$

$(x-7)(x+3) = 0$

$x = 7$ or $\cancel{x = -3}$ CHECK