2.1-2.2

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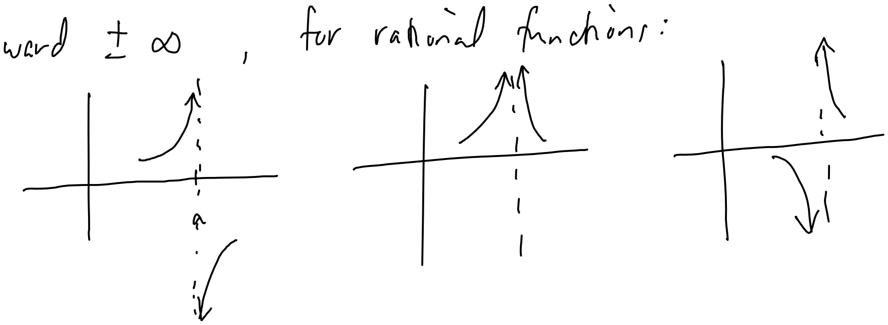
7:09 AM

§2.1 Rational Functions $f(x) = \frac{\text{poly}}{\text{poly}} = \frac{p(x)}{q(x)}$

domain is wherever $q(x) \neq 0$

if $q(a) = 0$ then $f(x)$ has either a hole at $x=a$
or a vertical asymptote at $x=a$.

Vertical Asymptote is a situation where the function values tend toward $\pm\infty$, for rational functions:



Ex. $f(x) = \frac{x^2+4}{x^2-1} = \frac{x^2+4}{(x-1)(x+1)}$, $x \neq \pm 1$

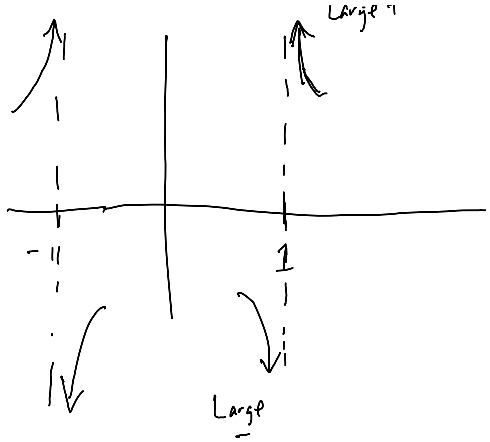
near $x=1$ what ~~f~~ values does f take on

near $x=1$, but to the left $\xrightarrow{\quad 1 \quad}$ x

$f(x) \longrightarrow \frac{\text{near } 5}{(\text{small neg})(\text{near } 2)} \rightarrow \frac{2.5}{\text{small} -} \rightarrow$ Large $-$

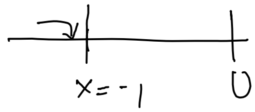
near $x=1$, but to the right $\xrightarrow{\quad 1 \quad}$ x

$f(x) \longrightarrow \frac{\text{near } 5}{(\text{small} +)(\text{near } 2)} \rightarrow \frac{2.5}{\text{small} +} \rightarrow$ Large $+$



near $x=-1$

to the left of $x=-1$



$$f(x)=\frac{x^2+4}{(x-1)(x+1)} \Rightarrow \frac{\text{near } 5}{(\text{near } -2)(\text{small } -)} \to \text{Large } +$$

to the right of $x=-1$

$$f(x)=\frac{x^2+4}{(x-1)(x+1)} \to \frac{\text{near } 5}{(\text{near } -2)(\text{small } +)} \to \text{Large } -$$

Ex. $f(x)=\dfrac{x^2-1}{x^2+4}$ domain is all reals, so no V.A's or holes

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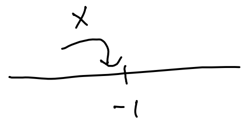
Ex. $g(x)=\dfrac{x^2+x}{x^2-1}$ domain is $x \neq \pm 1$

$$=\frac{x(x+1)}{(x-1)(x+1)},$$

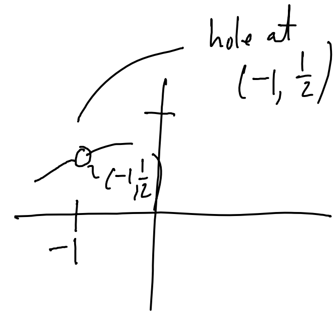
there is a VA at $x=1$ (you get the details)

what about at $x=-1$

near $x = -1$

to the left 

$$g(x) = \frac{x\cancel{(x+1)}}{(x-1)\cancel{(x+1)}} \to \frac{\text{near } -1}{\text{near } -2} = \frac{1}{2}$$



to the right

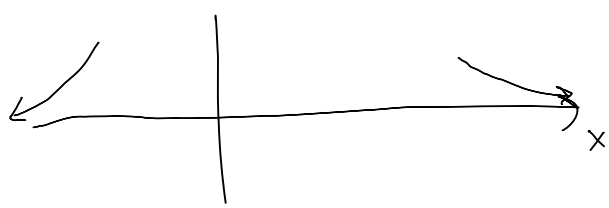
$$g(x) = \frac{x\cancel{(x+1)}}{(x-1)\cancel{(x+1)}} \to \frac{\text{near } -1}{\text{near } -2} = \frac{1}{2}$$

Horizontal Asymptotes: what can happen as x gets Large (+ or -) in a rational function

Can behave like poly's or can level off:

if x is large

ex. $f(x) = \frac{1}{x^2+5} \to \frac{1}{\text{Large}} \to$ small



$y = 0$ is Horizontal asymptote

(FACT: for rational functions: a horizontal asymptote is always the same in both directions as $x \to +\infty$ and as $x \to -\infty$)

ex. $f(x) = \frac{x^2}{2x^2+5} \to \frac{\text{large}}{\text{large}} \leftarrow$ CLUE to do something called rescaling.

as x gets large

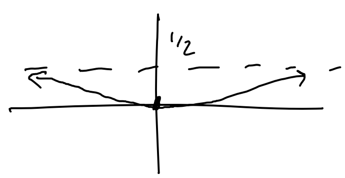
rescale = divide through by largest object (ignore constants)

$$f(x) = \frac{x^2}{2x^2+5} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \frac{\frac{x^2}{x^2}}{\frac{2x^2}{x^2} + \frac{5}{x^2}} = \frac{1}{2+\frac{5}{x^2}}$$

now as x gets large $\longrightarrow \dfrac{1}{2+\frac{5}{\text{large}}} \longrightarrow \dfrac{1}{2+\text{small}} \longrightarrow \dfrac{1}{2}$

$y = \frac{1}{2}$ is a horizontal asymptote of $f(x)$

because $f(x)$ gets close to $\frac{1}{2}$ as x gets large $(+ \text{ or } -)$



$\boxed{2^x \quad \text{vs} \quad x^2}$

§2.2 Exponentials

exponential function : $f(x) = b^x$, $b > 0$, $b \neq 1$

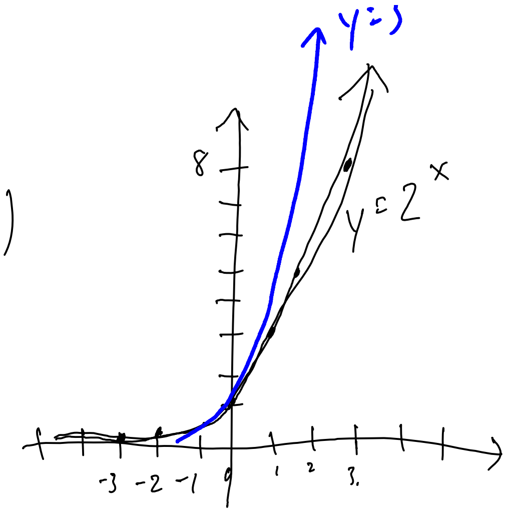
e.g. $f(x) = 2^x$ (base 2 exp.)

2^x

2^x

x	L
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$
4	$2^4 = 16$
-1	$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
-3	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

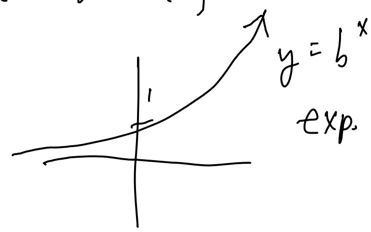
exp growth
(base 2 = doubling)



the domain ~~of~~ of an exp. is $(-\infty, \infty)$

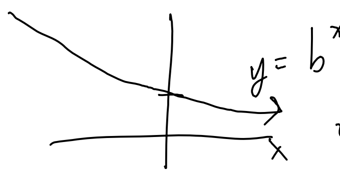
the range is $(0, \infty)$

if $b > 1$



exp. growth

if $0 < b < 1$



exp. decay.

$f(x) = \left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x}$ ($\leftarrow$ reflect graph of 2^x across y-axis)