t-test for a population mean μ (σ unknown)

What? Compare an unknown population mean, μ, to a hypothesized value, μ_0

When?
1. σ is *unknown*
2. population is normally distributed (in practice the t-test is fairly robust for non-normal distributions as long as the sample is reasonably large and/or the distribution isn't too "non-normal")
3. Simple Random Sample

How?
1. Define μ. Rewrite question sybolically in terms of μ.
2. Select appropriate null and alternative hypothesis (differ slightly from text)

null	$H_0 : \mu = \mu_0$	$H_0 : \mu = \mu_0$	$H_0 : \mu = \mu_0$
alternative	$H_a : \mu < \mu_0$	$H_a : \mu > \mu_0$	$H_a : \mu \neq \mu_0$
	left-tailed	right-tailed	two-tailed

3. Choose the level of significance, α according to your tolerance for a type I error. If a type I error would have bad consequences in your problem, then α may need to be smaller, but beware that a smaller value of α increases the probability of a type II error and also makes it more difficult to find a statistically significant H_a.
4. Check conditions!
5. Calculate the test statistic $t = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$
6. Determine the P-value (the probability, if H_0 were true, of observing a test-statistic at least as favorable for H_a ... this is more or less the probability of observing data like the sample data when H_0 is true)

$H_a : \mu < \mu_0$	$H_a : \mu > \mu_0$	$H_a : \mu \neq \mu_0$

7. Conclusions. If $P \leq \alpha$ reject H_0 in favor of H_a (the sample data is too rare if H_0 is true). If $P > \alpha$ then do not reject H_0 in favor of H_a (the sample data is not that unusual if H_0 is true, don't have enough evidence for H_a).

Example: *USA Today* reported that the state with longest mean life span is Hawaii, where the population mean life span is 77 years. A random sample of 20 obituary notices in the *Honolulu Advertizer* gave the following information about life span (in years) of Honolulu residents:

$$72 \quad 68 \quad 81 \quad 93 \quad 56 \quad 19 \quad 78 \quad 94 \quad 83 \quad 84$$

$$77 \quad 69 \quad 85 \quad 97 \quad 75 \quad 71 \quad 86 \quad 47 \quad 66 \quad 27$$

Test at $\alpha = 0.05$ to see if the mean population life span is actually less than 77 years.

χ^2-test for a population variance σ^2

What? Compare an unknown population variance, σ^2, to a hypothesized value, σ_0^2

When?
1. population is normally distributed (important - this test is sensitive to departures from normality)
2. Simple Random Sample

How?
1. Define σ^2. Rewrite question sybolically in terms of σ^2.
2. Select appropriate null and alternative hypothesis (differ slightly from text)

null	$H_0 : \sigma^2 = \sigma_0^2$	$H_0 : \sigma^2 = \sigma_0^2$	$H_0 : \sigma^2 = \sigma_0^2$
alternative	$H_a : \sigma^2 < \sigma_0^2$	$H_a : \sigma^2 > \sigma_0^2$	$H_0 : \sigma^2 \neq \sigma_0^2$
	left-tailed	right-tailed	two-tailed

3. Choose the level of significance, α.
4. Check conditions!
5. Calculate the test statistic $\chi^2 = \dfrac{s^2(n-1)}{\sigma_0^2}$
6. Determine the P-value

$H_a : \sigma^2 < \sigma_0^2$	$H_a : \sigma^2 > \sigma_0^2$	$H_0 : \sigma^2 \neq \sigma_0^2$

7. Conclusions.

Example: A new kind of typhoid shot is being developed by a medical research team. The old typhoid shot was known to protech the population for a mean time of 36 months, with a standard deviation of 3 months. To test the time variability of the new shot, a random sample of 23 people were given the new shot. Regular blood tests showed that sample standard deviation of protection times was 1.9 months. Using a 0.05 significance level test the claim that the new typhoid shot has a smaller variance of protection times.