Tuesday, October 20, 2009
1:13 PM

6. The serum cholesterol levels of a certain population of boys follow a normal distribution with a mean of 170 mg/dl and a standard deviation of 30 mg/dl.

(*a*) Find the probability that a randomly chosen boy has a serum cholesterol level of 155 mg/dl or less. $X \sim N(170, 30)$, $P(X < 155)$

(*b*) Find the percentage of boys with values between 125 mg/dl and 215 mg/dl.

(*c*) Find the probability that the mean serum cholesterol level of a random sample of 25 boys is below 182 mg/dl.

(*d*) Determine the probability that the mean serum cholesterol level of a random sample of 100 boys is below 164 mg/dl.

(c) consider $\bar{x}$'s for $n = 25$

describe the sampling distribution of the sample means

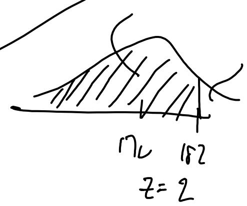
Ctr: $\mu_{\bar{x}} = \mu = 170$

spread: $\sigma_{\bar{x}} = \sigma/\sqrt{n} = \frac{30}{\sqrt{25}} = 6$

Shape: normal (because parent pop is normal)

$\bar{x} \sim N(170, 6)$

$P(\bar{x} < 182) =$



(d)

4.3 Confidence Intervals

given μ, σ → predict how $\bar{x}$'s behave (sampling distribution)

BACKWARDS

want to go the other direction (Inferential statistics)

given $\bar{x}, s, n$, etc. try to predict μ (or σ, etc).

include uncertainty from sampling variability.

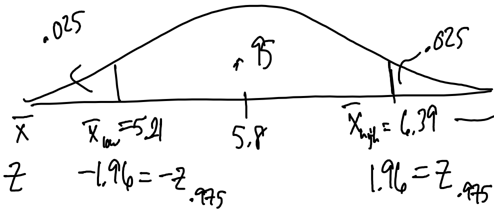
Warm Up bass weights (lbs) $= X \sim N(5.8, 1.2)$

For samples of size $n = 16$, find the "middle" 95% of sample mean $\bar{x}$

$\mu_{\bar{x}} = \mu = 5.8$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.2}{\sqrt{16}} = .3$

$\bar{x} \sim N(5.8, .3)$

$\bar{X} \sim N(5.8, .3)$



for 95% of samples of size n=16, the average bag weight will be b/w 5.21 lbs & 6.39 lbs.

$\bar{X}_{low} = \sigma_{\bar{X}} z + \mu_{\bar{X}}$

$= .3(-1.96) + 5.8 = 5.21$

$= \frac{\sigma}{\sqrt{n}}(-z_{.975}) + \mu$

$\bar{X}_{high} = \sigma_{\bar{X}} z + \mu_{\bar{X}}$

$= .3(1.96) + 5.8 = 6.39$

$= \frac{\sigma}{\sqrt{n}} z_{.975} + \mu$

$\mu \pm z_{.975} \frac{\sigma}{\sqrt{n}} \longleftarrow$ middle 95% of sample means

Casting a net or interval of width $\pm z_{.975} \sigma/\sqrt{n}$ around μ captures the 95% most probable sample means.

$\boxed{\text{Reverse Idea}}$: given an $\bar{X}$, cast a net around it w/ width $\pm z \, \sigma/\sqrt{n} \longrightarrow$ gives a plausible range of values for μ, we also get a confidence that reflects our confidence in the estimate

---

Level C confidence interval (CI) for μ – Z-interval

What? A plausible range of values for μ (accounts for sampling variability) PLUS a confidence level.

When? (1) pop. std. dev σ is known.
(2) pop. is normally dist. or $n \geq 30$.
(3) sample is a SRS.

How: $\boxed{\bar{X} \pm z_{1-\alpha} \frac{\sigma}{\sqrt{n}}}$

α is the error rate

C is [illegible]

How: $\boxed{\bar{X} \pm z_{1-\alpha/2} \frac{\sigma}{\sqrt{n}}}$

C is the confidence level

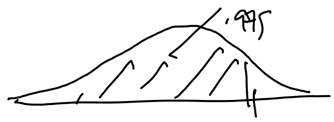
$1-\alpha$ is the confidence level

e.g. 95% confidence interval

use $\alpha = .05$

so $z_{1-\alpha/2} = z_{1-\frac{.05}{2}} = z_{.975}$

Confidence	α	$z_{1-\alpha/2}$
.9	.1	1.645
.95	.05	1.96
.99	.01	2.576



$\cancel{z_{.975}} = 1.96$

$z_{.975}$

Ch4: 2, 4, 7, 9